Workshop 4

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1 Goal

The goal of this workshop is to practice proving statements about arithmetic operations on sequences.

2 Questions

1. For which (if any) real numbers a and b does the sequence

$$\left\{(a^n + b^n)^{1/n}\right\}_{n=1}^{\infty}$$

converge? For those values, what does it converge to?

2. Find an example of each of the following, or prove that one cannot exist.

   (a) Sequences $\{x_n\}$ and $\{y_n\}$, neither of which converge, but such that the sum $\{x_n + y_n\}$ converges.

   (b) A sequence $\{x_n\}$ which converges and $\{y_n\}$ which does not converge such that $\{x_n + y_n\}$ converges.

   (c) A sequence $\{b_n\}$ that converges with $b_n \neq 0$ for all n, and such that $\{1/b_n\}$ does not converge.

   (d) A bounded sequence $\{a_n\}$ and a convergent sequence $\{b_n\}$ such that $\{a_n - b_n\}$ is bounded.

   (e) Two sequences $\{a_n\}$ and $\{b_n\}$ where both $\{a_n\}$ and $\{a_n b_n\}$ converge but $\{b_n\}$ does not converge.

3. In this problem, we look at Cesaro means:

   (a) Prove that if a sequence $\{s_n\}$ converges, then the sequence of averages

$$y_n = \frac{x_1 + \cdots + x_n}{n}$$

   also converges to the same thing.

   (b) Give an example to show that the sequence of averages can converge even if the x_n don't converge.

4. Let $\{a_n\}$ be a bounded (but not necessarily convergent) sequence of real numbers and let $\{b_n\}$ be a sequence that converges to 0. Prove that $a_n b_n$ converges to 0. Can we say anything about the sequence $a_n b_n$ if b_n converges to some nonzero number?