Math 311 Section 01 **Quiz 9** Spring 2018

Name: ______________________________

1. (1 point) Which of the following sets are *compact*? Select all that apply.
   - ○ $(0,1)$
   - $\surd$ $[0,\infty)$
   - ○ $(0,\infty)$
   - $\surd$ $[0,1]$
   - ○ $(0,1]$

2. (1 point) __**F**__ True or False: Every closed set is compact.

3. (1 point) __**T**__ True or False: A continuous function on a compact set is uniformly continuous.

4. (1 point) Complete the statement of the Heiene-Borel Theorem below:

   A subset of $\mathbb{R}$ is compact if and only if it is ____________________ and ____________________.

   Solution: A subset of $\mathbb{R}$ is compact if and only if it is **closed** and **bounded**.

5. (1 point) Determine the error in the following argument.

   Claim 1. *Let $f : \mathbb{R} \to \mathbb{R}$ be continuous at every $x \in \mathbb{R}$. Then f is uniformly continuous.*

   Proof. Let $\varepsilon > 0$. For every $x \in \mathbb{R}$, there is a δ_x such that for all $y \in \mathbb{R}$ with $|y - x| < \delta_x$, $\big|f(y) - f(x)\big| < \varepsilon$.
   Let $\delta = \min_{x\in\mathbb{R}} \delta_x$. Then, for all $x, y \in \mathbb{R}$ with $|x - y| < \delta$, we have $\big|f(x) - f(y)\big| < \varepsilon$. Hence, f is uniformly continuous. □

   Solution: There are an infinite number of δ_x, so the minimum may not exist. Furthermore, even if it does exist, there is no reason that it need be positive.