Name: ______________________________

1. (1 point) Which of the following sets are *closed*? Select all that apply.
   - ○ $(0, 1)$
   - √ $[0, \infty)$
   - ○ $(0, \infty)$
   - √ $[0, 1]$
   - ○ $(0, 1]$

2. (1 point) __**F**__ True or False: Every closed set is bounded.

3. (2 points) For what $a \in \mathbb{R}$ does the limit $\lim_{x \to a} \lfloor x \rfloor$ exist where $\lfloor x \rfloor$ is the greatest integer less than or equal to x? You do not need to provide a formal proof, but you must give some justification for your answer. (1 point for answer, 1 point for justification)

Solution: The limit exists if and only if $a \notin \mathbb{Z}$. At those points, the function jumps. (Drawing a graph is particularly helpful for this explanation)

4. (1 point) Determine the error in the following argument.

Claim 1. *Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers such that*

$$\lim_{n \to \infty} (a_n - b_n) = 0.$$

Then a_n and b_n converge to the same number.

Proof.

$$0 = \lim_{n \to \infty} (a_n - b_n) = \lim_{n \to \infty} (a_n) - \lim_{n \to \infty} (b_n).$$

Hence, by moving the last term to the left hand side, we have

$$\lim_{n \to \infty} (a_n) = \lim_{n \to \infty} (b_n)$$

□

Solution: We know that the limit of the sum (or difference) is the sum (or difference) of the limits *as long as all of the limits exist.* There is no assumption here that the limits need exist, so we cannot split the limit as was done in the first line of the proof.