Math 311 Section 01 | **Quiz 7** | Spring 2018

Name: ______________________________

1. (1 point) __**F**__ True or False: For every $S \subseteq \mathbb{R}$, either S is closed or S is open.
2. (1 point) __**F**__ True or False: An open set has no accumulation points.
3. (1 point) __**T**__ True or False: The complement of a closed set is always open.
4. (1 point) Define the set $S \subseteq \mathbb{R}$ as follows:

$$S = \bigcap_{x>0} (-x, x). \tag{1}$$

Is S open or closed? Give some reasoning for your claim, but it does *not* need to be a precise proof.

> **Solution:** $S = \{0\}$. Since every singleton set is closed, S is closed.

5. (1 point) Determine the error in the following argument:

Claim 1. *Let $n \in \mathbb{Z}$. If $n^2 + 2n + 4$ is divisible by 4, then n is even.*

Proof. Suppose $n \in \mathbb{Z}$ is even. Then there is an integer k such that $n = 2k$. So,

$$n^2 + 2n + 4 = 4k^2 + 4k + 4 = 4(k^2 + k + 1).$$

Since $k^2 + k + 1 \in \mathbb{Z}$, this shows that $n^2 + 2n + 4$ is divisible by 4. □

> **Solution:** This is a proof of the converse, which is *not* logically equivalent to the original statement.

6. (1 point (bonus)) Is the empty set open or closed? You do *not* need to prove your answer.

> **Solution:** The empty set is both open and closed. This is also true for $\mathbb{R}$. In fact, these are the only two sets which are both open and closed.