Calculate the Fourier cosine series of \( f(x) \) defined by \( f(x) = x \) for \( -\pi \leq x \leq \pi \).

We proceed by integration by parts:
\[
\int_{-\pi}^{\pi} \sin(n \pi x) \cos(n \pi x) \, dx = -\frac{\sin(n \pi x) \sin(n \pi x)}{n \pi} \bigg|_{-\pi}^{\pi}.
\]

Thus, we have:
\[
\int_{-\pi}^{\pi} f(x) \cos(n \pi x) \, dx = \frac{1}{n \pi} \left( \sin(n \pi x) \sin(n \pi x) \right)_{-\pi}^{\pi}.
\]

This gives us the general form for the coefficients:
\[
a_n = \frac{2}{\pi} \int_{0}^{\pi} x \cos(n \pi x) \, dx.
\]

To find the Fourier cosine series, we need to calculate the coefficients \( a_n \) for all \( n \geq 1 \).

For even values of \( n \), we have:
\[
a_n = \frac{2}{\pi} \int_{0}^{\pi} x \cos(n \pi x) \, dx = \frac{2}{\pi} \left( \frac{\sin(\pi n \pi x)}{n \pi} \right)_{0}^{\pi} = 0.
\]

For odd values of \( n \), we have:
\[
a_n = \frac{2}{\pi} \int_{0}^{\pi} x \cos(n \pi x) \, dx = \frac{2}{\pi} \left( \frac{-\cos(\pi n \pi x)}{n \pi} \right)_{0}^{\pi} = \frac{2}{n \pi}.
\]

Thus, the Fourier cosine series is:
\[
f(x) = \sum_{n=1}^{\infty} \frac{2}{n \pi} \cos(n \pi x).
\]

Now, assume that the Fourier cosine series of \( f(x) \) converges to \( f(x) \) at any point of the interval \( [0, \pi] \). Let us compute the limit:
\[
\lim_{n \to \infty} \int_{0}^{\pi} f(x) \cos(n \pi x) \, dx.
\]

We know that \( f(x) = x \) for \( x = 0 \), and in this case, the limit of the integral is zero.

To find the value of \( a_n \), we have:
\[
a_n = \frac{2}{\pi} \int_{0}^{\pi} x \cos(n \pi x) \, dx.
\]

For even values of \( n \), we have:
\[
a_n = \frac{2}{\pi} \left( \frac{\sin(\pi n \pi x)}{n \pi} \right)_{0}^{\pi} = 0.
\]

For odd values of \( n \), we have:
\[
a_n = \frac{2}{\pi} \left( \frac{-\cos(\pi n \pi x)}{n \pi} \right)_{0}^{\pi} = \frac{2}{n \pi}.
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Thus, the Fourier cosine series of \( f(x) \) is:
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Now, assume that the Fourier cosine series of \( f(x) \) converges to \( f(x) \) at any point of the interval \( [0, \pi] \). Let us compute the limit:
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Finally, we have:
\[
\int_{0}^{\pi} x \cos(n \pi x) \, dx = \frac{2}{\pi} \left( \frac{\sin(\pi n \pi x)}{n \pi} \right)_{0}^{\pi} = 0.
\]

And we have:
\[
\sin(\pi x) = \pi \sin(\pi x) \sin(x).
\]

Thus, we have:
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\]

with \( \sin(x) \), we have:
\[
\sin(\pi x) = \pi \sin(\pi x) \sin(x).
\]
We consider the system of equations:

\[ \begin{align*}
\frac{\partial x}{\partial t} &= f(x, t), \\
\frac{\partial y}{\partial t} &= g(x, t)
\end{align*} \]

with initial conditions:

\[ \begin{align*}
x(t_0) &= x_0, \\
y(t_0) &= y_0
\end{align*} \]

1. We want to show whether \( x(t) \) is an equilibrium of the problem:

We need to solve for \( x(t) \) when \( t_0 = 0 \) with \( x(t_0) = 0 \).

2. We now search for position equilibrium \( X = P = \beta \).

3. We now search for position equilibrium \( X = P = \beta \).

4. We now search for position equilibrium \( X = P = \beta \).

5. Now search for position equilibrium \( X = P = \beta \).

The graphical resolution shows that \( \beta \) is a solution of the equations, with \( \beta = \ldots \).

For each \( \beta \), find the line \( \beta = \ldots \).

The equilibrium of the problem \( \beta \) of the form \( x(t) = \beta \), \( \ldots \)

with \( \beta \in \mathbb{R} \), \( \ldots \), with the associated equilibrium:

\[ X(t) = \beta \]

We now investigate graphically:

\[ \beta \]

\[ x(t) = \ldots \]

\[ y(t) = \ldots \]
1) Let us compute a slice of the heat flow between a and b at t.

The total energy in the slice is:

\[ \frac{\partial E}{\partial t} = - \int_{f}^{b(t)} \phi(x) \, dx \]

and the energy leaving at time \( t \) is:

\[ \int_{a}^{b(t)} \frac{dE}{dt} \, dx \]

Thus,

\[ \frac{dE}{dt} = - \int_{f}^{b(t)} \phi(x) \, dx - \int_{a}^{b(t)} \frac{dE}{dt} \, dx \]

in flux.

2) Suppose that the equilibrium temperature does exist.

Then:

\[ \frac{dE}{dt} = - \int_{f}^{b(t)} \phi(x) \, dx \]

and there is an equilibrium with respect to \( x \).

Therefore, the sum of the work done on the system and the work done on the surroundings is zero.

We have:

\[ \int_{a}^{b(t)} \frac{dE}{dt} \, dx = 0 \]

the equilibrium temperature is \( \theta \).

3) We wish to find the equilibrium state of the system.

Thus:

\[ \frac{dE}{dt} = - \int_{a}^{b(t)} \phi(x) \, dx \]

and there is an equilibrium state.

We have:

\[ \int_{a}^{b(t)} \frac{dE}{dt} \, dx = 0 \]

the equilibrium temperature is \( \theta \).

4) Is \( \theta \) a real solution of the problem?

For \( \theta \) to be a real solution of the problem, there should exist \( x \) such that:

\[ \frac{dE}{dt} = - \int_{a}^{b(t)} \phi(x) \, dx \]

and:

\[ \int_{a}^{b(t)} \frac{dE}{dt} \, dx = 0 \]

Thus, \( \theta \) is a real solution of the problem.

5) For \( \theta = \beta^2 \) to be an equilibrium, one should have:

\[ \int_{a}^{b(t)} \frac{dE}{dt} \, dx = 0 \]

As in the previous case, there is no equilibrium at \( \theta = \beta^2 \).

6) For \( \theta = \beta^2 \), it is an equilibrium, and there should exist \( x \) such that:

\[ \int_{a}^{b(t)} \frac{dE}{dt} \, dx = 0 \]

there is no equilibrium at \( \theta = \beta^2 \).

7) For \( \theta = \beta^2 \), the temperature at \( T(t) \) associated to \( \phi(x) \) is:

\[ T(t) = \frac{\beta^2}{\phi(x)} \]

The solution for a general solution in the system reader:

\[ \frac{dE}{dt} = \sum_{i=1}^{n} \phi(x) \]

We obtain that \( \theta = \beta^2 \) is the temperature at \( x = 2 \text{ m} \) of the equilibrium state.

Thus, it is a possible state of the system, as it represents a possible equilibrium temperature.