Exercise 1 (A characterization of the supremum using limits).
Let $A$ be a subset of $\mathbb{R}$ which is non empty and bounded from above.

1. Why does $A$ admit a supremum?
2. Show that, for any $x \in \mathbb{R}$, $x = \sup(A)$ if and only if the following two properties hold:
   - $x$ is an upper bound for $A$,
   - for any $\varepsilon > 0$, there exists $a \in A$ such that $x - \varepsilon < a \leq x$.
3. By using the previous question, show that, for any $x \in \mathbb{R}$, $x = \sup(A)$ if and only if the following two properties hold:
   - $x$ is an upper bound for $A$,
   - there exists a sequence $(a_n)$ of elements of $A$ such that $a_n \rightarrow x$.

Exercise 2 (The ‘Sandwich Theorem’)
Let $(a_n)_{n \in \mathbb{N}}$, $(b_n)_{n \in \mathbb{N}}$ and $(c_n)_{n \in \mathbb{N}}$ be three sequences of real numbers satisfying the following inequality:

$$\forall n \in \mathbb{N}, \quad b_n \leq a_n \leq c_n.$$ 

Show that, if $(b_n)$ and $(c_n)$ converge to the same limit $\ell \in \mathbb{R}$, then $(a_n)$ also converges to $\ell$. 