Exercise 1
Let $a < b$ be two real numbers, and $c \in (a, b)$ be another point. Let $f : (a, b) \to \mathbb{R}$ be a function which is differentiable on $(a, c)$ and $(c, b)$, and such that:
\[
\forall x \in (a, b) \setminus \{c\}, \quad f'(x) > 0.
\]
(1) Show that $f$ does not admit any local extremum on $(a, b)$.
(2) Application: Show that the function $f : \mathbb{R} \to \mathbb{R}$, defined by $x \mapsto x^3$ does not have any local extremum on $\mathbb{R}$.

Exercise 2
Let $f : \mathbb{R} \to \mathbb{R}$ be a function which is differentiable at 0, and such that:
\[
\forall x, y \in \mathbb{R}, \quad f(x + y) = f(x)f(y).
\]
(1) Prove that $f$ is differentiable on $\mathbb{R}$, and that its derivative satisfies:
\[
\forall x \in \mathbb{R}, \quad f'(x) = f'(0)f(x).
\]
(2) Assuming the properties of the exponential function, infer that the function $f$ is actually:
\[
\forall x \in \mathbb{R}, \quad f(x) = e^{cx},
\]
where $c = f'(0)$. 