Abstracts

1 Arrábida July 2 –July 5, 2017

Matteo Bonforte (Universidad Autonoma de Madrid) matteo.bonforte@uam.es
Nonlinear and Nonlocal Degenerate Diffusions on Bounded Domains

ABSTRACT: We investigate quantitative properties of nonnegative solutions $u(t, x) \geq 0$ to the nonlinear fractional diffusion equation, $\partial_t u + \mathcal{L}F(u) = 0$ posed in a bounded domain, $x \in \Omega \subset \mathbb{R}^N$, with appropriate homogeneous Dirichlet boundary conditions. As $\mathcal{L}$ we can use a quite general class of linear operators that includes the three most common versions of the fractional Laplacian $(-\Delta)^s$, $0 < s < 1$, in a bounded domain with zero Dirichlet boundary conditions; many other examples are included. The nonlinearity F is assumed to be increasing and is allowed to be degenerate, the prototype being $F(u) = |u|^{m-1}u$, with $m > 1$,. We will shortly present some recent results about existence, uniqueness and a priori estimates for a quite large class of very weak solutions, that we call weak dual solutions. We will devote special attention to the regularity theory: decay and positivity, boundary behavior, Harnack inequalities, interior and boundary regularity, and asymptotic behavior. All this is done in a quantitative way, based on sharp a priori estimates. Although our focus is on the fractional models, our techniques cover also the local case s = 1 and provide new results even in this setting. A surprising instance of this problem is the possible presence of nonmatching powers for the boundary behavior: for instance, when $\mathcal{L} = (-\Delta)^s$ is a spectral power of the Dirichlet Laplacian inside a smooth domain, we can prove that, whenever $2s \geq 1 - 1/m$, solutions behave as $dist^{1/m}$ near the boundary; on the other hand, when $2s < 1 - 1/m$, different solutions may exhibit different boundary behaviors even for large times. This unexpected phenomenon is a completely new feature of the nonlocal nonlinear structure of this model, and it is not present in the elliptic case. The above results are contained on a series of recent papers in collaboration with A. Figalli, Y. Sire, X. Ros-Oton and J. L. Vazquez.

Eric Carlen (Rutgers University, USA) carlen@math.rutgers.edu
Quantum Master Equations

ABSTRACT: Quantum many particle systems have been studied in terms of quantum mater equations since the pioneering work of Pauli shortly after the introduction of quantum mechanics. The derivation of kinetic equations from such models is more recent, but began in the 1970's in a period of rapid development of the theory of quantum Markov semigroups. Lately classical Master equations describing many particle systems with binary interactions, such as the Kac Master equations have been – and continue to be – extensively developed. This talk presents

recent work done in collaboration with Carvalho and Loss, and will also include results from work with Wennberg and with Maas on problems related to quantum mater equations and quantum kinetic theory.

Raffaelle Esposito (M&MOCS-Universita dell'Aquila) raff.esposito@gmail.com
Hydrodynamic Limit of a Kinetic Gas Flow Past an Obstacle

ABSTRACT: Given a fixed obstacle and a non-zero small velocity at the infinity, we consider the classical fluid dynamic problem of the flow with zero velocity on the obstacle and non zero velocity at infinity. The fluid on the kinetic scale is described by the Boltzmann equation and the kinetic boundary conditions are given by the diffuse reflection with fixed wall temperature and zero mean velocity. We construct the unique steady Boltzmann solution flowing around such an obstacle with the prescribed velocity for large x. It approaches the corresponding incompressible Navier-Stokes steady flow, as the mean-free path goes to zero. Furthermore, we establish the error estimate between the Boltzmann solution and its Navier-Stokes approximation. Our method consists of new L^6 and L^3 estimates in the unbounded exterior domain to be associate with an L^2-L^∞ strategy, as well as an iterative scheme preserving the positivity of the distribution function.

Maria del Mar Gonzalez (Universidad Autonoma, Madrid) mariamar.gonzalezn@uam.es
Gluing methods for the fractional Yamabe problem with isolated singularities

ABSTRACT: We construct solutions for the fractional Yamabe problem with isolated singularities. When the singular set is just one point, a Delaunay type solution may be constructed by solving a non-local ODE. In the case of a finite number of singularities, we succesfully use a gluing method despite the nonlocality of the problem. We believe that this method can be applied in many other non-local situations. This is joint work with Weiwei Ao, Azahara DelaTorre and Jucheng Wei.

Maria Infusino (Univ. Konstanz, Germany) maria.infusino@uni-konstanz.de
The infinite dimensional moment problem as a new approach to realizability

ABSTRACT: The realizability problem is a fundamental question which naturally appears in the analysis of complex systems, but which is still open in many of its aspects. This talk aims to introduce a new interpretation of this problem as an infinite dimensional version of the classical moment problem, which has recently proved to be an effective approach towards new advances in this area. I will focus on the full moment problem on a special class infinite dimensional subsets of the space $D'(\mathbb{R}^d)$ of generalized functions on $\mathbb{R}^d$ ($d \in \mathbb{N}$), which is general enough to encompass several instances of the realizability problem. The question addressed is whether an infinite sequence of symmetric generalized functions is actually the moment sequence of a finite non-negative Borel measure supported on a given closed subset K of $D'(\mathbb{R}^d)$. In a recent joint work with T. Kuna and A. Rota we provide necessary and sufficient solvability conditions bearing only on the starting sequence in the case when K is defined by polynomial constraints on $D'(\mathbb{R}^d)$. Our result combines a classical theorem about the moment problem on nuclear spaces with some techniques recently developed for the moment problem on basic semi-algebraic sets of R^d, providing solvability conditions which can be more easily verified than the ones known so far in literature. Moreover, I will introduce some concrete examples of subsets of generalized functions to which our result applies e.g. the set of all point configurations and outline some new directions we are currently investigating.

Michael Loss (Georgia Tech, USA) loss@math.gatech.edu
Entropy decay for the Kac master equation

ABSTRACT: The Kac master equation models the behavior of a large number of randomly colliding particles. Due to its simplicity it allows, without too much pain, to investigate a number of issues. E.g., Mark Kac, who invented this model in 1956, used it to give a simple derivation of the spatially inhomogeneous Boltzmann equation. One important issue is the rate of approach to equilibrium, which can be analyzed in various ways, using, e.g., the gap or the entropy. Explicit entropy estimates will be discussed for a Kac type master equation modeling the interaction of a finite system with a large but finite reservoir. This is joint work with Federico Bonetto, Alissa Geisinger and Tobias Ried.

Rossana Marra (Univ. of Rome "Tor Vergata") marra@roma2.infn.it
Stationary solutions to the Boltzmann equation and their hydrodynamic limit

ABSTRACT: The stationary solutions to the Bolzmann equation, despite their relevance in applications, are much less studied than the initial value problem and no general existence theory is yet available. When suitable external forces or boundary conditions are included, interesting non equilibrium solutions arise. The main subject of the talk is the study of stationary solutions of the Boltzmann equations, both for finite and small Knudsen number, in a general domain in contact with a non-homogeneous thermal reservoir. Based on new constructive coercivity estimates for both steady and dynamic cases, we construct a unique non-negative solution, continuous away from the grazing set and exponentially asymptotically stable. Finally, we are able to answer to the longstanding open problem of the rigorous derivation of the steady incompressible Navier-Stokes-Fourier system from the Boltzmann theory, in the presence of a small external field and a small boundary temperature variation for the diffuse boundary condition. Indeed, by using a new L^2-L^∞ approach and new regularity-gain procedure (a new L^6 estimate), we prove that, near the global equilibrium, there is a unique positive solution to the stationary Boltzmann equation converging to the stationary solution to the incompressible Navier-Stokes-Fourier system. Moreover the solution is exponentially stable for small initial perturbations.

Sara Merino Aceituno (Imperial College London) s.merino-aceituno@imperial.ac.uk
A new flocking model through body attitude coordination

ABSTRACT: We present a new model for multi-agent dynamics where each agent is described by its position and body attitude: agents travel at a constant speed in a given direction and their body can rotate around it adopting different configurations. Agents try to coordinate their body attitudes with the ones of their neighbors. This model is inspired by the Vicsek model. The goal of this talk will be to present this new flocking model, its relevance and the derivation of the macroscopic equations from the particle dynamics. In collaboration with Pierre Degond (Imperial College London), Amic Frouvelle (Universit Paris Dauphine) and Ariane Trescases (University of Cambridge).

Maria Joao Oliveira (Univ. Aberta, Portugal) mjoliveira@fc.ul.pt
A new approach to the combinatorial harmonic analysis on configuration spaces

ABSTRACT: A procedure for lifting polynomials on $\mathbb{R}$ to polynomials on the space of distributions $\mathcal{D}'(\mathbb{R}^d)$ is discussed. Using this procedure we recover, in particular, the Hermite, Charlier and

Laguerre polynomials on $\mathcal{D}'(\mathbb{R}^d)$ and define new classes of polynomials. Among them, the infinite dimensional analog of the falling factorials, that is, the K-transform. Recall that the K-transform is a mapping of combinatorial type, which determines the duality between point processes on $\mathbb{R}^d$ and their correlation measures. Properties of the K-transform previously unknown are discussed.

Mario Pulvirenti (Univ. of Rome "la Sapienza") pulviren@mat.uniroma1.it
From N-Body Schroedinger to Hartree: Uniformity in $\hbar$

ABSTRACT: In this talk I approach the following problem. When dealing with the derivation of the Hartree equation from the N-Body Schroedinger equation, the estimate of the error are usually strongly diverging in $\hbar$. On the other hand, for classical systems, the convergence from the Newton equations to Vlasov is well established so that one may expect estimates independent of $\hbar$, at least for suitable initial states. I discuss this problem based on a work in collaboration with F. Golse and T. Paul.

Xavier Ros-Oton (University of Texas, Austin) ros.oton@math.utexas.edu
Free boundary regularity in the parabolic fractional obstacle problem

ABSTRACT: We study the regularity of the free boundary in the parabolic obstacle problem for the fractional Laplacian. This problem arises in American option models when the assets prices are driven by pure jump Lvy processes.

Our main result establishes that, when $s > 1/2$, the free boundary is a $C^{1,\alpha}$ graph in x and t near any regular free boundary point. Furthermore, we also prove that solutions are C^{1+s} in x and t near such points, with a precise expansion.

Ana Jacinta Soares (Universidade do Minho) ajsoares@math.uminho.pt
From the kinetic theory formulation of chemical processes to reaction-diffusion equations

ABSTRACT: Processes involving diffusion and chemical reactions appear in many physical and biological systems. The problem of deriving hydrodynamic equations from kinetic models to describe such processes has gain a great attention in the mathematical community. In this talk we present some contributions arising in this direction starting from a kinetic model of simple reacting spheres for a mixture with four components undergoing a reversible chemical reaction of bimolecular type. The presentation is focused on the mathematical modeling of reactive mixtures and formal derivation of reaction-diffusion equations of Maxwell-Stefan type.

Juan Luis Vazquez (Universidad Autonoma de Madrid) juanluis.vazquez@uam.es
Linear and nonlinear diffusion with nonlocal fractional operators

ABSTRACT: This is a report on progress made by the author and collaborators on the topic of nonlinear diffusion equations involving long distance interactions in the form of fractional Laplacian operators. Special attention to asymptotic patterns is given.

2 FCUL, July 5 –July 7, 2017

Eric Carlen (Rutgers University, USA) carlen@math.rutgers.edu

Introduction to Operator inequalities, Non-commutative Probability and Quantum Statistical Mechanics

ABSTRACT: This talk provides an elementary introduction to some problems in the intersection of the three active research areas listed in the title. The focus will be on explaining some ideas involving the analysis of "large deviations" that have been very successfully applied to problems in classical statistical mechanics, and indeed, much of this methodology has been developed in the context of investigating such problems. For quantum statistical mechanics, the non-commutativity of observables introduces many challenges, and a full quantum analog of the mathematical framework that has proven so useful in the classical case is missing. We shall give a simple survey of several possible ways forward that are the focus of current research.

Raffaelle Esposito (M&MOCS-Universita dell'Aquila) raff.esposito@gmail.com
Hydrodynamic limit of steady non equilibrium solutions to the Boltzmann equation

ABSTRACT: We discuss the problem of the hydrodynamic limit of the Boltzmann equation with emphasis on the steady non equilibrium solutions. We review some older results based on the famous Hilbert expansion and boundary layer expansion. Then we present some more recent developments avoiding the boundary layer expansion.

Maria Infusino (Univ. Konstanz, Germany) maria.infusino@uni-konstanz.de
The realizability problem for point processes

ABSTRACT: The problem of describing a point process from its low order correlation functions is well-known in the classical theory of fluids as a realizability problem, but naturally occurs also in several other areas dealing with the analysis of complex systems. In this talk I will give an introduction to this p roblem and then consider the special case of translation invariant point processes on the d-dimensional lattice ($d \in N$). In particular, I will focus on the question of establishing whether two given functions $\rho_1(i)$ and $\rho_2(i,j)$ non-negative and symmetric on $\mathbb{Z}^d$ are the first two correlation functions of at ranslation invariant point process on $\mathbb{Z}^d$. If such a process exists then the correlation functions can be written as $\rho_1(i) = \rho \in \mathbb{R}^+$ and $\rho_2(i,j) = \rho^2 g(i-j)$ with $g : \mathbb{Z}^d \to \mathbb{R}^+$. Therefore, the main challenge is to find the maximal value of the uniform density ρ for which realizability can be established for a prescribed g. I will review some explicit constructions of realizing processes when g has a specific form particularly relevant for the study of heterogenous materials in chemistry, iso-g processes, and when $d = 1$. Finally, I will present an explicit construction of an iso-g process on $\mathbb{Z}^d$ for any $d \geq 2$ recently developed together with E. Caglioti and T. Kuna. Our construction is the first to produce a lower bound for the maximal realizable density which improves the general lower bounds already known in literature and so provides a better approximation of the feasibility region for this class of realizability problems.

Michael Loss (Math. Dept. Georgia Tech, USA) loss@math.gatech.edu
The Kac master equation: Propagation of chaos, derivation of Boltzmann equation and approach to equilibrium

ABSTRACT: In this talk we shall review a number of issues concerning the Kac master equation which models the behavior of a large number of randomly colliding particles. I shall explain the notion of propagation of chaos and derivation of the spatially inhomogeneous Boltzmann equation.

Likewise we will have a closer look at the rate of approach to equilibrium, using the gap and the entropy. This review is based mostly on joint work with Eric Carlen and Maria Carvalho.

Mario Pulvirenti (Univ. of Rome la Sapienza, Italy) pulviren@mat.uniroma1.it
The mechanism of chaos for classical particle systems: Propagation in time of statistical independence, scaling limits and kinetic equations (Vlasov, Boltzmann, Landau)

ABSTRACT: In this talk I describe the mechanism of chaos for a classical particle system. This fundamental property is the propagation in time of the statistical independence, if it is initially postulated. This allows to pass from a N-particle description to a single equation for the one-particle distribution, driven by an effective kinetic equation. I discuss how this concept, under suitable scaling limits, provides a derivation (sometimes mathematically justified) of the most popular kinetic equations as the one of Vlasov, Boltzmann and Landau