Practice Test One B, Math 292, 2018

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1. Find the general solution of

$$t^3 x'(t) + t^2 x(t) - x^2(t) = 2t^4$$

for $t > 0$, and the corresponding flow transformation.

2. Consider the two equations

$$\textbf{I} \quad (y')^2 + y^2 = 1 \qquad \text{and} \qquad \textbf{II} \quad (y')^2 - y^2 = 1 \ .$$

One has a unique solution with $y(t) = y_0 \in (-1, 1)$, and the other has infinitely many such solutions. Which one is which? Justify your answer.

3. Let $\mathbf{v}(x, y) = (xy + 12, x^2 + y^2 - 25)$. Find all equilibrium points of $\mathbf{v}$, and determine which, if any, are Lyapunov stable, asymptotically stable, or unstable whenever this can be determined by linearization. Justify your answer, and sketch the flow curves near each equilibrium point.

4. Let $A = \begin{bmatrix} 3 & 1 \\ -1 & 5 \end{bmatrix}$ Let $\mathbf{f}(t) = (1, t)$. Find the solution of

$$\mathbf{x}'(t) = A\mathbf{x}(t) + \mathbf{f}(t)$$

with $\mathbf{x}(0) = (1, 1)$.

5. Define the matrices

$$M = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \quad \text{and} \quad A = 3\begin{bmatrix} 7 & 8 \\ 8 & 10 \end{bmatrix} \ .$$

(a) Find a positive matrix $M^{1/2}$ such that $(M^{1/2})^2 = M$.

(b) Find a matrix K such that if $\mathbf{x}(t)$ and $\mathbf{y}(t)$ are related by $\mathbf{y}(t) = M^{1/2}\mathbf{x}(t)$, then $\mathbf{x}(t)$ solves $M\mathbf{x}''(t) = -A\mathbf{x}(t)$ if and only if $\mathbf{y}(t)$ solves $\mathbf{y}''(t) = -K\mathbf{y}(t)$.

(c) Find all values of $\omega > 0$ such that the solutions of

$$M\mathbf{x}''(t) = -A\mathbf{x}(t) + \cos(\omega t)(1, 1)$$

have resonance.