Homework Assignment 6, Math 292, Spring 2018

Eric A. Carlen[1]
Rutgers University

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1. (a) Find the general solution of the homogeneous differential equation

$$t^2x'' + 3tx' + x = 0 \ .$$

(b) Find the general solution to the equation

$$t^2x'' + 3tx' + x = \ln(t)$$

(c) Find the solution of

$$t^2x'' + 3tx' + x = \ln(t)$$

that satisfies $x(1) = 0$ and $x(3) = 0$, or explain why there is o such solution.

2. (a) The equation

$$xu'' - u' + (1-x)u = 0 \tag{0.1}$$

has one solution of the form $u_1(x) = e^{\alpha x}$. Find this solution and a second independent solution $u_2(x)$.

(b) Find functions p and q so that with $\mathcal{L}u = (pu')' + q$, $\mathcal{L}u = 0$ if and only if $xu'' - u' + (1-x)u = 0$.

(c) For the Sturm-Liouville operator $\mathcal{L}$ from part **(b)**, find the Green's function for solving $\mathcal{L}u = f$ subject to $u(1) = u(2) = 0$, or explain why it does not exist. Finally, find the solution of $\mathcal{L}u = x^2$ subject to $u(1) = u(2) = 0$.

(d) Solve $\mathcal{L}u = x^2$ subject to Neumann boundary conditions on $[1,2]$.

3: Solve the equation

$$\frac{\partial}{\partial t}h(x,t) = (1+x)^2\frac{\partial^2}{\partial^2 x}h(x,t)$$

for $x \in (0,1)$, $t > 0$, subject to

$$h(0,t) = h(1,t) = 0$$

and

$$h(x,t) = \sin\left(\pi\frac{\ln(1+x))}{\ln(2)}\right) \ .$$

4: (a) Consider the equation $\mathcal{L}u = 0$ where

$$\mathcal{L}u = \left(\frac{1}{x}u'\right)' + \frac{1-2x}{x^2}u \ .$$

(a) Are there any solutions of this equation that have more than one zero on $[1, \infty)$? Justify your answer: First find an equation of the form

$$y'' + V(x)y(x)$$

so that every solution of this equation on $(0, \infty)$ is a nonzero multiple of a solution of $\mathcal{L}u = 0$, and apply the Strum oscillation theorems.

(b) Does $\mathcal{L}u = f(x)$ have a solution satisfying $u(1) = u(2) = 0$ for all continuous $f(x)$ on $[1, 2]$? Justify your answer.