Homework Assignment 5, Math 292, Spring 2018

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1. Consider the boundary value problem

$$x^2u''(x) - 2u(x) = \frac{3}{x} \quad \text{with} \quad u(1) = u(2) = 0 \ .$$

(a) Find the equivalent Liuoville form $\mathcal{L}u = f$ of of this problem.

(b) Find the Green's function for $\mathcal{L}u = f$ with $u(1) = u(2) = 0$, and solve the boundary value problem posed at the beginning.

2. Consider the boundary value problem

$$xu''(x) - (x+2)u'(x) + 2u(x) = x^3e^x \quad \text{with} \quad u(1) = u(2) = 0 \ .$$

(a) Find the equivalent Liuoville form $\mathcal{L}u = f$ of of this problem.

(b) Find the Green's function for $\mathcal{L}u = f$ with $u(1) = u(2) = 0$, and solve the boundary value problem posed at the beginning. (In this case, you might want to simply use the "unsimplified" formula since the solutions of the homogeneous equation that satisfy $u_1(1) = 0$ and $u_2(2) = 0$ will be complicated enough that the "simplification" is lost.)

3. Consider the boundary value problem

$$x^2u''(x) - 6u(x) = x^4 \quad \text{with} \quad u(1) = u(2) = 0 \ .$$

(a) Find the equivalent Liuoville form $\mathcal{L}u = f$ of of this problem.

(b) Find the Green's function for $\mathcal{L}u = f$ with $u(1) = u(2) = 0$, and solve the boundary value problem posed at the beginning.

4. Consider the boundary value problem

$$x^2u''(x) + xu'(x) - 4u(x) = x \quad \text{with} \quad u(1) = u(2) = 0 \ .$$

(a) Find the equivalent Liuoville form $\mathcal{L}u = f$ of of this problem.

(b) Find the Green's function for $\mathcal{L}u = f$ with $u(1) = u(2) = 0$, and solve the boundary value problem posed at the beginning.

5. Find the general solution of

$$u''(x) - u'(x) - 6u(x) = e^x \quad \text{with} \quad u'(0) = u'(1) = 0 \ .$$

Notice that in this problem we have imposed a boundary condition on the derivative $u'(x)$ instead of on $u(x)$.

5. Find the general solution of

$$(x^2 + x)u''(x) + (2 - x^2)u'(x) - (2 + x)u(x) = x(x + 1)^2 \quad \text{with} \quad u'(1) = u'(2) = 0 \ .$$

Notice that in this problem we have imposed a boundary condition on the derivative $u'(x)$ instead of on $u(x)$.