Homework Assignment 1, Math 292, Spring 2016

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January 20, 2018

1. (a) 1. Find the general solution of

$$tx'(t) = x(t) + 3t^2 \ .$$

(b) Find the flow transformation $\Phi_{t,t_0}(x)$ specified by this equation.

(c) Find the particular solution $x(t)$ that satisfies $x(1) = 2$.

2. (a) Find the general solution of

$$x'(t) + \frac{1}{3}x(t) = e^t x^4(t) \ .$$

(b) Find the particular solution $x(t)$ that satisfies $x(1) = 2$. Over what time interval $t \in (a, b)$ is the solutions a continuously differentiable function?

3. (a) Find the general solution of

$$tx'(t) = tx^2(t) - x(t) - \frac{1}{t} \ .$$

(b) For any (x_0, t_0) with $t_0 > 0$, find the solution $x(t)$ of this equation that satisfies $x(t_0) = x_0$

(c) Write down a formula for the flow transformation Φ_{t,t_0} generated by this equations. Verify explicitly that $\Phi_{3,2}(\Phi_{2,1}(x)) = \Phi_{3,1}(x)$ for all x.

4. Consider the equation

$$x'(t) = 2t\frac{t^2 + x(t)}{t^2 - x(t)} \ . \tag{1}$$

This is not first order linear, Bernoulli or Ricatti, and it is not separable as it stands. But introducing a new variable, we can get a more amenable equation. Introduce $y(t)$ through

$$y(t) = \frac{x(t)}{t^2}$$

for $t > 0$.

(a) Show that $x(t)$ solves (1) for $t > 0$ if and only if $y(t)$ solves a separable equation for $t > 0$.

(b) Find the solution of (2) with $y(1) = y_0$, $y_0 \neq -1$.

(c) Find the solution of (1) with $x(1) = x_0$, $x_0 \neq -1$.

5. Find the general solution of the equation

$$tx''(t) = 1 + (x'(t))^2 \ .$$