Practice Test II, Math 291 Fall 2107

November 28, 2017

NAME: ______________________________

INSTRUCTIONS: Show all computations. Points of credit are assigned to the various stages of computation which must all be recorded explicitly. Correct answers without supporting computations receive zero credit. Show your work. Finally, if the answer is, for example, $\sqrt{2}$, write this. Do not give decimal approximation such as 1.414213562: all questions have exact answers; give them in exact form.

1. Let $f(x,y) = x^3y + x^2y + x$.

(a) Find all of the critical points of f. Evaluate the Hessian matrix of f at each of these critical points, and determine where each is a local maximum, a local minimum, a saddle, or undecidable from the Hessian.

(b) Sketch a contour plot of f in the vicinity of each critical point. **Especially here, show the computations that lead to the plot to get credit**.

(c) Find all unit vectors $\mathbf{u}$ in $\mathbb{R}^2$ such that that

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} f((1,0) + t\mathbf{u})\bigg|_{t=0} = 0 \ .$$

2. Use the method of Lagrange multipliers in both parts of this problem.

(a) Let D be the region consisting of all points (x, y) satisfying

$$x^2 \leq y \leq 3 + 2x \ .$$

Let $f(x, y) = xy - 3x$. Find the minimum and maximum values of f on D, and find all minimizers and maximizers.

(b) Find the point on the paraboloid

$$z = 3 - \frac{1}{2}((x-1)^2 + (y-1)^2)$$

that is closest to the origin. **Hint:** The cubic polynomial $t^3 - 3t^2 + t + 2$ has a simple integer root.

3. **(a)** Let D be the set in the positive quadrant of $\mathbb{R}^2$ that is bounded by

$$x^2 + y^2 = 4 \qquad \text{and} \qquad xy = 1 \ .$$

Let $f(x, y) = \sqrt{1 + x^2 + y^2}$. Compute $\int_D f(x, y)\mathrm{d}A$.

(b) Let D be the set in $\mathbb{R}^2$ that is given by

$$x^2 \leq y \leq 2x^2 \qquad \text{and} \qquad x^3 \leq y \leq 2x^3 \ .$$

Let $f(x, y) = \dfrac{x}{y}$. Compute $\int_D f(x, y)\mathrm{d}A$.

4. **(a)** Let $\mathcal{S}$ be the part of the graph of $z = xy$ that lies above the graph of $z = x^2 + y^2 - 4$. Compute the area of $\int_{\mathcal{S}}$.

Extra Credit: The function $f(x, y, z) = 2x^3y^2 - 3x^2y - 3y + x^2 + y^2 + z^2$ has $(1, 1, 1)$ as one of its critical points. Is this a local minimum, a local maximum, a saddle, on a critical point whose nature cannot be decided by analysis of the Hessian of f? Does the function have either minimum or a maximum on all of $\mathbb{R}^3$?