Homework 9, Math 291 Fall 2017

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1. Let D be the triangle bounded by the three lines $y = x$, $y = 2x$ and $y = 3x-1$. Let $f(x, y) = xy$. Compute

$$\int_D f(x, y)\mathrm{d}A \ .$$

2. Let D be the region in the plane that is outside the unit circle, and to the left of the parabola $x = 5/4 - y^2$. Let $f(x, y) = x^2 + y^2$.

$$\int_D f(x, y)\mathrm{d}A \ .$$

3. Let D be the set in $\mathbb{R}^2$ that is given by

$$x^2 \le y \le 2x^2 \qquad \text{and} \qquad x^3 \le y \le 2x^3 \ .$$

Let $f(x, y) = \dfrac{x}{y}$. Compute $\int_D f(x, y)\mathrm{d}A$.

4. Let D be the region in upper right quadrant of $\mathbb{R}^2$ that is inside the circle $x^2 + y^2 = 1$, and between the parabolas $y = 2x^2$ and $y = 3x^2$. Compute $\int_D xy\mathrm{d}A$.

5: (a) Let D be the set in the positive quadrant of $\mathbb{R}^2$ that bounded by

$$\begin{aligned} y &= x \\ y &= \sqrt{3}x \\ y &= x^2 + y^2 \end{aligned}$$

Let $f(x, y) = \sqrt{1 + x^2 + y^2}$. Compute $\int_D f(x, y)\mathrm{d}A$.

6. Let D be the region in the upper right quadrant between the curves

$$x = \frac{1}{y^2} \qquad \text{and} \qquad x = \frac{4}{y^2}$$

and between the curves

$$y = x^2 \qquad \text{and} \qquad y = 4x^2 \ .$$

Compute $\int_D (x^2 + y^2)\mathrm{d}A$.

7. Let $\mathcal{V}$ be the region in $\mathbb{R}^3$ that is inside the cylinder

$$x^2 + y^2 = y$$

and bounded above and below by

$$z = x^2 + y^2 \qquad \text{and} \qquad z = \sqrt{x^2 + y^2}\ .$$

Compute the volume of this region.

8. (a) Let $\mathcal{V}$ be the region in $\mathbb{R}^3$ that lies below the graph of $z = 1 - x^2$, and above the graph of $z = y^2$. Compute the volume of $\mathcal{V}$.

9. Let $\mathcal{V}$ be the region in $\mathbb{R}^3$ that lies inside the sphere $x^2 + y^2 + z^2 = 4$, and above the graph of $z = 1/\sqrt{x^2 + y^2}$. Compute the volume of $\mathcal{V}$ **and** the total surface area of its boundary. (There are two pieces to the boundary.)

10. (a) Let $\mathcal{S}$ be the surface the is the part of the graph of $z = 1 - \sqrt{x^2 + y^2}$ that lies inside the cylinder $(x-1)^2 + y^2 = 1$, whihc is a cylider of radius 1 running parallel to the z-axis. Let $f(x, y, z) = z$ Compute $\int_{\mathcal{S}} f \mathrm{d}S$. **Hint:** Write the equation $(x-1)^2 + y^2 = 1$ in polar coordinates.

11. Let $\mathcal{S}$ be the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the plane $x + z = 1$. Compute $\int_{\mathcal{S}} f(x, y, z) \mathrm{d}S$ where $f(x, y, z) = y/\sqrt{x^2 + y^2}$. To get full credit, carry the computations through to the point that only an integral over a single variable remains to be evaluated.