# Homework 8, Math 291 Fall 2017

Eric A. Carlen[1]
Rutgers University

November 12, 2017

**1:** Consider the function $f(x,y) = -xy^2 + 3y + x^2y$.

**(a)** Find all critical points, and compute the Hessian matrix at each crtical point.

**(b)** Determine whether each critical point is a local minimum, a local maximum, a saddle, or if the Hessian is degenerate at the critical point.

**(c)** Sketch a contour plot of $f$ in the vicinity of each critical point.

**2:** Consider the function $f(x,y) = \dfrac{x^2y}{1+2x^4+y^4}$.

**(a)** Find all critical points in the open upper right quadrant; i.e., the set of $(x,y)$ with $x,y > 0$. Compute the Hessian matrix at each such crtical point.

**(b)** Determine whether each such critical point is a local minimum, a local maximum, a saddle, or if the Hessian is degenerate at the critical point.

**(c)** Sketch a contour plot of $f$ in the vicinity of each such critical point.

**3:** For any real number $a$, let $A(a)$ be the matrix

$$A(a) := \begin{bmatrix} 3 & 1 & a \\ 1 & 2 & a \\ a & a & 1 \end{bmatrix} .$$

For which values of $a$ are all of the eigenvlaues of $A(a)$ strictly positivle? Justify your answer to recieve credit.

**4:** Let $a > b > 0$ be given. Consider the circle in the $x,y$ plane in $\mathbb{R}^3$ parameterized by $a(\cos u, \sin u, 0)$, $0 \le u < 2\pi$. The set $\mathcal{S}$ of all points in $\mathbb{R}^3$ whose distance from this circle is exactly $b$ is torus. It is parameterized by

$$\mathbf{X}(u,v) = a(\cos u, \sin u, 0) + b(\cos u \sin v, \sin u \sin v, \cos v) = (\cos u(a + b\sin v)\ ,\ \sin u(a+b\sin v)\ ,\ b\cos v)\ ,$$

where $0 \le u, v < 2\pi$.

**(a)** Check that $\mathbf{X}_u(u,v)$ and $\mathbf{X}_v(u,v)$ are linearly independent for all $u, v$.

**(b)** Compute the equation for the tangent plane to the torus at $\mathbf{X}(\pi/4, \pi/3)$.

**(c)** Compute the principle curvatures at $\mathbf{X}(\pi/4, \pi/3)$, and find an orthonormal basis of eigenvectors of the tangent plane at $\mathbf{X}(\pi/4, \pi/3)$ that consists of eigenfunctions of the shape operator.

**(d)** Compute the Gaussian curvature of the torus at each point $\mathbf{X}(u,v)$.