Homework 7, Math 291 Fall 2017

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1: Let $f(x,y) = xy + 2x - 2y$. Find the minimum and maximum values of f on the region of $\mathbb{R}^2$ that lies below the parabola $y = 1 - x^2$ and above the x-axis, $y = 0$.

2: The intersection of the plane $2z = x + 9$ and the cone $z^2 = x^2 + y^2$ is an ellipse. Find the points on the ellipse that are closest to and furthest from the origin.

3: Let $f(x,y) = x + yz$. Consider the curve given by the pair of equations $3x^2 + 4y^2 + z^2 = 1$ and $z = x^2 + y^2$.

(a) Write down the system of three equations in three unknowns that must be satisfied by any maximizers of f on the curve, according to Lagrange's Theorem.

(b) One of the equations may be used to eliminate z from this system of three equations, resulting in a system of two equations in the two variables x and y. Find this system. That is, eliminate z.

(c) Draw graphs, by any means, showing the solution sets of the two equations on the x,y plane, Count the points of intersection an determine the number of solutions. Then pick a starting point near each solution, and run one step of Newton's method to get an improved approximate solution.

4: Find an orthonormal basis of $\mathbb{R}^2$ consisting of eigenvectors for the matrix $A = \begin{bmatrix} 5 & 9 \\ 9 & -4 \end{bmatrix}$.

5: Find an orthonormal basis of $\mathbb{R}^3$ consisting of eigenvectors for the matrix $A = \begin{bmatrix} 4 & 0 & 2 \\ 0 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix}$.

6: Consider the function $f(x,y) = xy^2 - \frac{1}{4}x^4 - \frac{1}{2}y^4$.

(a) Find all critical points, and compute the Hessian matrix at each critical point.

(b) Determine whether each critical point is a local minimum, a local maximum, a saddle, or if the Hessian is degenerate at the critical point.

(c) Sketch a contour plot of f in the vicinity of each critical point.