Homework 5, Math 291 Fall 2015

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1: Let $f(x,y)$ be given by

$$f(x,y) = -xy^2 + 3y + x^2y \ .$$

(a) Find all critical points of f.

(b) Find the equation of the tangent plane to the graph of $z = f(x,y)$ at $(x,y) = (2,1)$.

(c) Suppose the positive y-axis runs due North, and the positive x-axis runs due East. If $z = f(x,y)$ represents the altitude at the point (x,y), and you pour out a glass of water at $(2,1)$, in what compass direction does the water run?

(d) Find all points (x,y) such that the normal vector to the tangent plane at (x,y) is a multiple of $(1,1,1)$.

2: Let $f(x,y)$ be given by

$$f(x,y) = xy^2 - x^4 - y^4 \ .$$

(a) Find all critical points of f.

(b) Find the maximum value of f over the whole plane, if there is one, and in that case, find all maximizers of f. Justify your answer.

(c) Find the minimum value of f over the whole plane, if there is one, and in that case, find all maximizers of f. Justify your answer.

3: Let $f(x,y)$ be given by

$$f(x,y) = \frac{x^2y}{1+x^4+y^4} \ .$$

(a) Find all critical points of f.

(b) Find the maximum value of f over the whole plane, if there is one, and in that case, find all maximizers of f. Justify your answer.

(c) Find the minimum value of f over the whole plane, if there is one, and in that case, find all maximizers of f. Justify your answer.

4: Let $f(x,y)$ be given by

$$f(x,y) = \frac{x^2y + y^2}{1+x^4+y^2} \ .$$

(a) Compute equations for the tangent planes to the graph of $z = f(x, y)$ at $(1, 1)$ and at $(-1, -1)$.

(b) These planes intersect in a line. Find a parameterization of the line.

5: Let $f(x, y)$ and $g(x, y)$ be given by

$$f(x, y) = -yx + 1 + x^2 \quad \text{and} \quad g(x, y) = x^2 + (y/4)^2 - 1 \ .$$

(a) How many solutions of the system

$$\begin{aligned} f(x, y) &= 0 \\ g(x, y) &= 0 \end{aligned}$$

are the in the upper-right quadrant? (The upper right quadrant is the set $\{(x, y : \ x, y \geq 0\}$.

(b) Take $bx_0 = (2, 1)$, and run one iteration of Newton's method to find a better approximate solution $\mathbf{x}_1$. Compute $f(\mathbf{x}_1)$ and $g(\mathbf{x}_1)$, and compare $\sqrt{f(\mathbf{x}_1)^2 + g(\mathbf{x}_1)^2}$ with $\sqrt{f(\mathbf{x}_0)^2 + g(\mathbf{x}_0)^2}$.