Homework 4, Math 291 Fall 2015

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1: Let $f(x,y)$ be given by

$$f(x,y) = \begin{cases} \dfrac{y\sin(xy)}{x^2+y^4} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0)\ . \end{cases}$$

At which points $(x_0, y_0) \in \mathbb{R}^2$ is the function f continuous? Justify your answer.

2: Let $f(x,y)$ and $g(x,y)$ be given by

$$f(x,y) = \begin{cases} \dfrac{x^2y^3}{x^4+y^6} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases} \qquad \text{and} \qquad g(x,y) = \begin{cases} \dfrac{x^5}{x^4+y^6} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}.$$

(a) Is the function f continuous at $(0,0)$? Justify your answer.

(b) Is the function g continuous at $(0,0)$? Justify your answer.

3: Let $f(x,y)$ and $g(x,y)$ be given by

$$f(x,y) = \begin{cases} \dfrac{x^2y}{x^4+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases} \qquad \text{and} \qquad g(x,y) = \begin{cases} \dfrac{x^2y^2}{x^4+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}.$$

(a) Is the function f continuous at $(0,0)$? Is is bounded on the closed unit disc $\{(x,y) \,:\, x^2+y^2 \leq 1\}$? Justify your answers.

(b) Is the function g continuous at $(0,0)$? Is is bounded on the closed unit disc $\{(x,y) \,:\, x^2+y^2 \leq 1\}$? Justify your answer.

4: Let $f(x,y)$ be a differentiable function on $\mathbb{R}^2$ such that $f(0,0) = 0$. Define a function $g(x,y)$ by

$$g(x,y) = \begin{cases} \dfrac{f(x,y)}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0)\ . \end{cases}$$

Suppose that f is continuously differentiable. Is it then necessarily the case that g is continuous? Justify your answer to receive credit.