

# PROOFS OF SOME RAMANUJAN SERIES BY THE WZ-METHOD

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ABSTRACT. The usual proofs of Ramanujan-type series for  $1/\pi$  are based on the use of modular equations. Instead, from 2002 we have succeeded in proving a few of them in an elementary way, by using the Wilf-Zeilberger (WZ)-method. For example, in 2010 we found a WZ-pair which proves

$$\sum_{n=0}^{\infty} \left(\frac{-1}{16}\right)^n \frac{(\frac{1}{2})_n (\frac{1}{3})_n (\frac{2}{3})_n}{(1)_n^3} (51n + 7) = \frac{12\sqrt{3}}{\pi},$$

where  $(a)_n = a(a+1)\cdots(a+n-1)$  and  $(a)_0 = 1$ . In this talk, we will not only give the WZ-pairs, but in addition we will show how to discover them. We will also see that the same method allows us to prove some formulas for  $1/\pi^2$  belonging to a new family which we discovered in 2002, and which has no modular origin. Then, we will conjecture more examples of series of the same type and discuss which of them (if not all) are WZ-provable. Finally, following another strategy we will use our WZ-pairs to derive some related hypergeometric identities.

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