This book is concerned with the study in two dimensions of stationary solutions $u_\varepsilon$ of a complex valued Ginzburg-Landau equation involving a small parameter $\varepsilon$. Such problems are related to questions occurring in physics: e.g., phase transition phenomena in superconductors and superfluids. The parameter $\varepsilon$ has a dimension of a length which is usually small. Thus, it is of great interest to study the asymptotics as $\varepsilon$ tends to zero.

One of the main results asserts that the limit $u_*$ of minimizers $u_\varepsilon$ exists. Moreover, $u_*$ is smooth except at a finite number of points called defects or vortices in physics. The number of these defects is exactly the Brouwer degree — or winding number — of the boundary condition. Each singularity has degree one — or as physicists would say, vortices are quantized.

The singularities have infinite energy, but after removing the core energy we are led to a concept of finite renormalized energy. The location of the singularities is completely determined by minimizing the renormalized energy among all possible configurations of defects.

The limit $u_*$ can also be viewed as a geometrical object. It is a minimizing harmonic map into $S^1$ with prescribed boundary condition $g$. Topological obstructions imply that every map $u$ into $S^1$ with $u=g$ on the boundary must have infinite energy. Even though $u_*$ has infinite energy, one can think of $u_*$ as having "less" infinite energy than any other map $u$ with $u=g$ on the boundary.

The material presented in this book covers mostly recent and original results by the authors. It assumes a moderate knowledge of nonlinear functional analysis, partial differential equations, and complex functions. This book is designed for researchers and graduate students alike, and can be used as a one-semester text.