Venue: The 6th Conference Room of Jingshi Hotel

Scientific Committee:
Kung-Ching Chang (co-chair), Ta-Tsien Li (co-chair)
Jiguang Bao, Yanyan Li, Yiming Long, Gang Tian

Organizers:
Yanyan Li (co-chair), Kaishun Wang (co-chair)
Haigang Li, Zhongwei Tang, Jingang Xiong, Xiaojing Xu

Invited Speakers:
Kung-Ching Chang (PKU) Yannick Sire (JHU)
Jianchun Chu (Northwestern) Gang Tian (PKU)
Peter Constantin (Princeton) Juan Luis Vázquez (U. Aut. Madrid)
Huabin Ge (Beijing Jiaotong) Vlad Vicol (NYU)
Changfeng Gui (UT San Antonio) Zhi-Qiang Wang (Utah State)
Tianling Jin (HKUST) Juncheng Wei (UBC)
Ta-Tsien Li (Fudan) Jingang Xiong (BNU)
Zenghu Li (BNU) Paul Yang (Princeton)
Yiming Long (Nankai) Jiangong You (Nankai)
Petru Mironescu (U. Lyon) Po-Lam Yung (CUHK)
Luc Nguyen (Oxford) Feng Zhou (ECNU)
Itai Shafrir (Technion)

Contact Us:
Email: jx@bnu.edu.cn (Jingang Xiong)
Phone: +86-18046512521

Supported By:
School of Mathematical Sciences, Beijing Normal University
International Conference on Partial Differential Equations and Applications

In Honor of 75th Birthday of Prof. Haïm Brezis

Scientific Committee:
Kung-Ching Chang (co-chair), Ta-Tsien Li (co-chair)
Jiguang Bao, Yanyan Li, Yiming Long, Gang Tian

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Sponsor:
School of Mathematical Sciences, Beijing Normal University

Beijing Normal University
July 1-3, 2019
Venue:

Conference Room 6 on the third floor, Jingshi Hotel

Invited Speakers:

- Kung-Ching Chang (PKU)
- Yannick Sire (JHU)
- Jianchun Chu (Northwestern)
- Gang Tian (PKU)
- Huabin Ge (Beijing Jiaotong)
- Zhi-Qiang Wang (Utah State)
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- Yiming Long (Nankai)
- Po-Lam Yung (CUHK)
- Petru Mironescu (U. Lyon)
- Feng Zhou (ECNU)
- Itai Shafrir (Technion)

Registration:

- June 30 11:30-20:30, the lobby of Jingshi Hotel on the first floor
- July 01 08:00-08:50, Conference Room 6 on the third floor

Contact Info:

Email: jx@bnu.edu.cn (Jingang Xiong)
Phone: 18046512521

Logistics:

Shan Chen, Zixiao Liu, Xuzhou Yang, Binbin Zheng
# Program

## July 1, morning

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<td>09:30-09:55</td>
<td>Opening ceremony &amp; Conference photo</td>
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<tr>
<td>10:00-10:45</td>
<td>Nonsmooth critical point theory and applications to the spectral graph theory</td>
<td>Kung-Ching Chang (Peking University)</td>
</tr>
<tr>
<td>11:00-11:45</td>
<td>Asymptotic synchronization of second order evolution systems</td>
<td>Ta-Tsien Li (Fudan University)</td>
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<td>12:00-14:00</td>
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## July 1, afternoon

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<td>14:00-14:45</td>
<td>Relative volume comparison along Ricci flow</td>
<td>Gang Tian (Peking University)</td>
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<tr>
<td>14:55-15:40</td>
<td>Minimizers of a variational problem for nematic liquid crystals with variable degree of orientation in two dimensions</td>
<td>Itai Shafrir (Technion)</td>
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<td>15:40-16:00</td>
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<tr>
<td>16:00-16:45</td>
<td>Dynamical System Approach to Spectral Theory of Quasi-Periodic Schrödinger Operators</td>
<td>Jiangong You (Nankai University)</td>
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<tr>
<td>16:55-17:40</td>
<td>On the rigidity and realizations of 3-dimensional hyperbolic polyhedrons</td>
<td>Huabin Ge (Beijing Jiaotong University)</td>
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<td>18:00-20:00</td>
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<tr>
<td>Time</td>
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<tr>
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<td>Iteration theories for periodic orbits, old and new</td>
<td>Yiming Long</td>
</tr>
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<td>Borderline asymptotics in Sobolev embedding</td>
<td>Zhi-Qiang Wang</td>
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<td>Type II Blow-up for Fujita Equation in the Matano-Merle Exponent Regime</td>
<td>Juncheng Wei</td>
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<td>12:00-14:00</td>
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<tr>
<td>14:00-14:45</td>
<td>Probabilistic models of stochastic population systems</td>
<td>Zenghu Li (Beijing Normal University)</td>
</tr>
<tr>
<td>14:55-15:40</td>
<td>$C^{1,1}$ regularity of geodesics of singular Kahler metrics.</td>
<td>Jianchun Chu</td>
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<tr>
<td>15:40-16:00</td>
<td>Tea Break</td>
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<tr>
<td>16:00-16:45</td>
<td>New developments in sphere covering inequality</td>
<td>Changfeng Gui (University of Texas at San Antonio)</td>
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<tr>
<td>16:55-17:40</td>
<td>Asymptotic behavior for higher order conformally invariant equations with isolated singularities</td>
<td>Tianling Jin (The Hong Kong University of Science and Technology)</td>
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<tr>
<td>18:00-20:00</td>
<td>Banquet</td>
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### July 3, morning

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<tr>
<th>Time</th>
<th>Topic</th>
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<tbody>
<tr>
<td>09:00-09:45</td>
<td>The Sobolev quotient of CR structures in dimension three</td>
<td>Paul Yang (Princeton University)</td>
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<td>10:00-10:45</td>
<td>Lifting in Sobolev spaces of manifold-valued maps</td>
<td>Petru Mironescu (Université de Lyon)</td>
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<tr>
<td>11:00-11:45</td>
<td>The Chang-Wilson-Wolff inequality and applications</td>
<td>Po-Lam Yung (The Chinese University of Hong Kong)</td>
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<td>12:00-14:00</td>
<td>Lunch</td>
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### July 3, afternoon

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<td>Half-harmonic maps, minimal surfaces with free boundary and Ginzburg-Landau approximation</td>
<td>Yannick Sire (Johns Hopkins University)</td>
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<tr>
<td>14:55-15:40</td>
<td>Optimal boundary regularity of weak solutions of the fast diffusion equations in domains</td>
<td>Jingang Xiong (Beijing Normal University)</td>
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<tr>
<td>15:40-16:00</td>
<td>Tea Break</td>
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<tr>
<td>16:00-16:45</td>
<td>Isolated singularities for some nonlinear elliptic equations and applications</td>
<td>Feng Zhou (East China Normal University)</td>
</tr>
<tr>
<td>18:00-20:00</td>
<td>Dinner</td>
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A special talk by Professor Haïm Brezis:

From Optimal Transport to the Plateau problem via Liquid crystals

**Time:** July 4, 09:00-10:00

**Room:** New Main Building 1124, Beijing Normal University

**Abstract.** I will discuss two proofs of the celebrated Monge-Kantorovich theorem in discrete Optimal Transport (OT). One of them is extremely elementary, self-contained, and can be understood by beginners. I will then describe an application to Liquid Crystals, which provides an explicit formula for the least energy required to produce a configuration with assigned defects. Next I will present striking connections that we recently discovered with P. Mironescu between OT and least area formulas for the classical Plateau problem.
Titles and Abstracts

Nonsmooth critical point theory and applications to the spectral graph theory
Kung-Ching Chang
Peking University

Abstract: Existing critical point theories including metric and topological critical point theories are difficult to be applied to some concrete problems in particular polyhedral settings, because the notions of critical sets could be either very vague or too large. To overcome these difficulties, we develop critical point theory for Lipschitzian functions defined on convex polyhedrons incorporated and mixed together with the stratified structure of polytopes and the decomposition of tangent cones. This yields natural extensions of classical results in critical point theory, such as the deformation theorem, mountain pass theorem, Liusternik Schnirelmann multiplicity theorem, the Lagrange multiplier theorem, etc. More importantly, eigenvectors for some eigenvalue problems involving graph 1-Laplacian coincide with critical points of the corresponding functions on polytopes, which indicates that the critical point theory proposed in the present paper can be applied to study the nonlinear spectral graph theory.

C^{1,1} regularity of geodesics of singular Kahler metrics
Jianchun Chu
Northwestern University

Abstract: In this talk, I will first describe a joint work with Valentino Tosatti and Ben Weinkove on optimal C^{1,1} regularity of geodesics of Kahler metrics. Next, I will show the optimal C^{1,1} regularity of geodesics of Kahler metrics on compact Kahler varieties away from the singular locus. This is a joint work with Nicholas McCleerey. A key step is to establish the boundary estimate for the complex Monge-Ampere equation that does not require strict positivity of the reference form near the boundary.

On the rigidity and realizations of 3-dimensional hyperbolic polyhedrons
Huabin Ge
Beijing Jiaotong University
Abstract: The topic is original from Cauchy's rigidity theorem and Steinitz's realization theorem of 3-dimensional Euclidean convex polyhedrons. Cauchy's rigidity theorem says that if two convex polyhedrons $P$ and $P'$ are combinatorically equivalent with corresponding facets being congruent, then $P$ is congruent to $P'$, while Steinitz's realization theorem says that any 3-connected planar graph can be realized as a convex polyhedron.

In this talk, we will give some new results on the rigidity of hyperbolic polyhedrons, we will also build some connections between realizations of hyperbolic polyhedrons and combinatorial Ricci flows on surfaces.

New developments in sphere covering inequality

Changfeng Gui
*University of Texas at San Antonio*

Abstract: In this talk, I will review the Sphere Covering Inequality and present some new forms of the inequality, including a version with singular terms on more general surfaces as well as a dual version. I will also show their applications in the study of singular Liouville-type problems with super harmonic weights. In particular, new uniqueness results for solutions of the singular mean field equation both on spheres and on bounded domains will be discussed. Some new symmetry results for the spherical Onsager vortex equation will also be presented.

Asymptotic behavior for higher order conformally invariant equations with isolated singularities

Tianling Jin
*The Hong Kong University of Science and Technology*

Abstract: We prove sharp blow up rates of solutions of higher order conformally invariant equations in a bounded domain with an isolated singularity, and show the asymptotic radial symmetry of the solutions near the singularity. This is an extension of the celebrated theorem of Caffarelli-Gidas-Spruck for the second order Yamabe equation with isolated singularities to higher order equations. Our approach uses blow up analysis for local integral equations, and is unified for all critical elliptic equations of order smaller than the dimension. We also prove the existence of Fowler solutions to the global equations, and establish a sup*inf type Harnack inequality of Schoen for integral equations.
Asymptotic synchronization of second order evolution systems

Ta-Tsien Li
Fudan University

Abstract: Under Kalman's criterion and suitable gap condition on the coupling matrices, the unique continuation of a complex system of elliptic operators can be reduced to the observability of a scalar equation. Based on this idea, we establish the strong stability and the asymptotic synchronization by groups for the corresponding evolution systems. This work shows that as the approximate boundary synchronization, Kalman's criterion still plays an important role for the strong stability and the asymptotic synchronization.

Probabilistic models of stochastic population systems

Zenghu Li
Beijing Normal University

Abstract: Branching processes are mathematical models of the evolution of stochastic population systems. The books by Athreya and Ney (1972) and Harris (1963) contain a lot about classical branching processes and their applications. Measure-valued branching processes over abstract spaces were introduced by Watanabe (1968), who proved those processes arose as high-density limits of branching particle systems. The connection of the processes with stochastic evolution equations was first investigated by Dawson (1975). The study has been stimulated from different subjects including stochastic partial differential equations, nonlinear partial differential equations, random trees and graphs. The developments have also led to better understanding of results in those subjects thanks to the works of Aldous (1991, 1993), Dynkin (1994, 2002), Le Gall (1999) and many others. We present a brief review of the study in branching processes focusing on its interplay with stochastic partial differential equations and nonlinear (quasi-linear) partial differential equations.

Iteration theories for periodic orbits, old and new

Yiming Long
Nankai University

Abstract: In this talk, I shall give a survey on the iteration theories for periodic solution orbits of various problems, from 1950s to now with some developments in recent years, including some of their applications to periodic solution problems.
Lifting in Sobolev spaces of manifold-valued maps

Petru Mironescu
Université de Lyon

Abstract: Let \( \pi: E \to \mathbb{N} \) be a Riemannian covering map, with \( \mathbb{N} \) compact, and let \( B^N \) be the unit ball in \( \mathbb{R}^N \). Given \( s > 0 \) and \( 1 < p < \infty \), the lifting problem in the Sobolev space \( W^{s,p}(B^N, \mathbb{N}) \) consists of deciding whether all maps \( u \in W^{s,p}(B^N, \mathbb{N}) \) can be lifted in \( W^{s,p} \), i.e., if we can write every such \( u \) in the form \( u = \pi \circ \varphi \), for some \( \varphi \in W^{s,p}(B^N, E) \). In general, the answer is negative. We discuss several aspects of this problem: nature of obstructions to lifting, substitutes to lifting, absence of a priori estimates and profile of "bad" \( u \)'s. We also present the complete solution to the lifting problem. Based in part on a joint work with Jean Van Schaftingen, and on previous contributions of Bourgain and Brezis, Bethuel and Chiron.

Minimizers of a variational problem for nematic liquid crystals with variable degree of orientation in two dimensions

Itai Shafrir
Technion

Abstract: We study the asymptotic behavior, when \( k \to \infty \), of the minimizers of the energy \( G_k(u) = \int_{\Omega}((k-1)^2|\nabla u|^2 + |\nabla u|^2) \), over the class of maps \( u \in H^1(\Omega, \mathbb{R}^2) \) satisfying the boundary condition \( u = g \) on \( \partial \Omega \), where \( \Omega \) is a smooth, bounded and simply connected domain in \( \mathbb{R}^2 \) and \( g: \partial \Omega \to S^1 \). The motivation comes from a simplified version of Ericksen model for nematic liquid crystals. We will present similarities and differences with respect to the analog problem for the Ginzburg-Landau energy. This is a joint work with Dmitry Golovaty.

Half-harmonic maps, minimal surfaces with free boundary and Ginzburg-Landau approximation

Yannick Sire
Johns Hopkins University

Abstract: I will report on some recent works related to the construction of minimal surfaces with free boundary, using an approach based on half-harmonic maps introduced by Da Lio and Riviere. This approach is rather flexible and gives a new point of view on this old problem both on the elliptic and parabolic sides. I will first describe the regularity theory of half-harmonic maps, motivated both by geometric applications and analytical
developments. I will then move towards the Ginzburg-Landau approximation of those maps, answering a question raised by Da Lio and Riviere in their first work. Tools involve Geometric Measure Theory, Compensation-compactness and a crucial role is played by monotonicity formulae.

**Relative volume comparison along Ricci flow**

Gang Tian  
*Peking University*

**Abstract:** In this talk, I will discuss my work with Z.L. Zhang on volume estimates along Ricci flow. We obtained a relative volume comparison result which extends Perelman’s famous non-collapsing result and Bishop-Gromov volume comparison for manifolds with Ricci curvature bounded from below.

**Borderline asymptotics in Sobolev embedding**

Zhi-Qiang Wang  
*Utah State University*

**Abstract:** We report recent work on the asymptotic behavior of extremal functions for the Sobolev embedding from $H^1$ into $L^p$ as $p \to 2$. This gives a precise convergence result of the ground states of the scalar field equation, and in turn yields a direct proof of the logarithmic Sobolev inequality by passing limit in the Sobolev embedding. This is a joint work with Chengxiang Zhang.

**Type II Blow-up for Fujita Equation in the Matano-Merle Exponent Regime**

Juncheng Wei  
*University of British Columbia*

**Abstract:** I will discuss various Type II blow-up phenomena for the classical Fujita equation $u_t = \Delta u + |u|^{p-1}u$ when $p$ belongs to Matano-Merle regime: $\frac{n+2}{n-2} < p < p_{\text{M}}(n)$. 
Optimal boundary regularity of weak solutions of the fast diffusion equations in domains

Jingang Xiong
Beijing Normal University

Abstract: We show boundary $C^{2,1}$ regularity and thus optimal higher order regularity for bounded positive weak solutions of the fast diffusion equations in smooth domains. This $C^{2,1}$ to the boundary regularity problem was raised by Berryman-Holland 1980. Previously, the interior smoothness and boundary Holder regularity were proved by Chen-DiBenedetto 1988 and DiBenedetto-Kwong-Vespri 1991. Our proof uses a geometric type structure, namely, a curvature-like evolution equation. This is joint with T. Jin.

The Sobolev quotient of CR structures in dimension three

Paul Yang
Princeton University

Abstract: Under the condition that the CR Paneitz operator be non-negative, we were able to prove the positivity of mass when the CR conformal Laplacian is positive. We studied the mass for a family of CR structures called the Rossi sphere, and show that the mass is negative when the structure is sufficiently close to the standard sphere. In addition, we showed that the Sobolev quotient is never attained in this case.

Dynamical System Approach to Spectral Theory of Quasi-Periodic Schrödinger Operators

Jiangong You
Nankai University

Abstract: The spectral theory of quasiperiodic operators is a fascinating field which continuously attracts a lot of attentions for its rich background in quantum physics as well as its rich connections with many mathematical theories and methods. In this talk, I will briefly introduce the problems in this field and their connections with dynamical system. I will also talk about some recent results joint with Avila, Ge, Leguil, Zhao and Zhou on both spectrum and spectral measure by reducibility theory in dynamical systems.
The Chang-Wilson-Wolff inequality and applications

Po-Lam Yung
The Chinese University of Hong Kong

Abstract: We will discuss an inequality about martingales due to Alice Chang, J.M. Wilson and Tom Wolff, and give some recent applications of the inequality to problems in harmonic analysis involving curvature. Some recent joint work with Shaoming Guo, Joris Roos and Andreas Seeger will be described.

Isolated singularities for some nonlinear elliptic equations and applications

Feng Zhou
East China Normal University

Abstract: In this talk, we discuss the isolated singular solutions for nonlinear elliptic equations, in particular for some problems involving Hardy-Leray potential. We present some suitable distributional identity for solutions, including with singularities locating on the boundary. We address some applications on nonexistence of some nonhomogeneous problems with the Hardy-Leray potentials and nonexistence of principle eigenvalue with indefinite potentials. The talk is based on joint works with H.Y. Chen, A. Quaas and R. Peng.