Comparison of the Wave & Heat Equations [Str 2.5.1, 2.5.4]

1. Show that there is no maximum principle for the wave equation.

2. (Optional) Let \( u(x, t) \) solve the wave equation on the whole line with bounded second derivatives. Define

\[
v(x, t) = \frac{c}{4\pi kt} \int_{-\infty}^{\infty} e^{-s^2c^2/4kt} u(x, s) ds.
\]

Show \( v(x, t) \) solves the heat equation (!) and that \( \lim_{t \to 0} v(x, t) = u(x, 0) \).

Hint: Write the formula as \( v(x, t) = \int H(s, t) u(x, s) ds \), where \( H(x, t) \) solves the diffusion equation with constant \( k/c^2 \) for \( t > 0 \). There differentiate \( v(x, t) \). You may need to use the fact that \( H(s, t) \) is essentially the source function of the diffusion equation with the spatial variable \( s \).

Vibrating Membrane & Bessel Functions [Hab, Sections 4.5, 7.7, Str 10.2.3]

3. Read Haberman, section 4.5, pp. 149-151

4. Read Haberman, section 7.7, pp. 303-315

5. Haberman, pp. 315-317, #7.7.1, 7.7.2d, 7.7.10,

6. (Optional) Read Haberman, section 7.8, pp. 318-325

7. (Optional) Read Strauss, section 10.2, pp. 264-270

8. (Optional) Suppose that you have a circular drum with wave speed \( c_d \) and radius \( a \) and a violin string with wave speed \( c_v \) and length \( L \). In order to make the fundamental frequencies of the drum and the violin the same, how would you choose the length?

Semi-Infinite Strings & Reflections, Method of Characteristics for Finite Strings [Hab, Sections 12.4, 12.5]

9. Read Haberman, sections 12.4-12.5, pp. 552-560

10. (Optional) Haberman, pp. 555-557, #12.4.1 – 12.4.8

11. (Optional) Haberman, pp. 560, #12.5.1 – 12.5.4