640:152 Calculus II, Midterm Exam #2, Spring 2012

Department of Mathematics, Rutgers University

NAME (PLEASE PRINT):

SIGNATURE:

ID #:

INSTRUCTOR:

SECTION:

- **Turn off/put away all mobile phones, computers, iPods, etc.**
- There is **one** question in Part I consisting of 2 problems and **six** multiple-part questions in Part II.
- Show answers and arguments in the space provided. You may use the back of the pages also, but indicate clearly any such material that you want marked. Answers given without supporting work may receive **zero credit**.
- This is a closed book exam. No calculators. No formula sheets.

Question	Points	Score
PART I		
1	10	
PART II		
2	18	
3	18	
4	18	
5	18	
6	18	
TOTAL	100	

GOOD LUCK!

PART I

1. The parts of this problem are not related.

   (a) Determine the limit $\lim_{n\to\infty} \sqrt{n}\ln\left(1+\frac{1}{n}\right)$, showing all work.

   (b) For what values of a does the integral $\int_2^\infty \frac{1}{x(\ln x)^a}\,dx$ converge? Justify your answer.

PART II

2. The parts of this problem are related.

   (a) Show that for $|x| < 1$,

$$\operatorname{arctanh} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots = \sum_{n=1}^{\infty} \frac{x^{2n-1}}{2n-1}.$$

   Hint: Recall that $\frac{d}{dx} \operatorname{arctanh} x = \frac{1}{1-x^2}$.

(b) Using your result from part (a), calculate the definite integral

$$\int_0^1 x^2 \operatorname{arctanh} \tfrac{x}{3} \, dx.$$

3. The parts of this problem are NOT related.

(a) Give a counterexample to show that the following statement is false: If the general term a_n of a sequence tends to zero, then $\sum_{n=1}^{\infty} a_n = 0$.

(b) Find a formula for the partial sum S_N of the series $\sum_{n=1}^{\infty}(-1)^n$ and show that the series diverges.

(c) Suppose that $S = \sum_{n=1}^{\infty} a_n$ is an infinite series with partial sum $S_N = 5 - \frac{2}{N^2}$. What is the value of a_3?

4. Determine whether the following series are absolutely convergent, only conditionally convergent, or divergent. Justify all answers and state any definitions or tests you use.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1/2}}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{4}}{n^2}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}3^n}{n^2 2^n}$.

5. The parts of this problem are NOT related.

(a) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^6 + n^2 + 1}}{n^4 - n + 3}$ converges.

(b) Expand the function $\frac{5x}{1+3x^2}$ as a power series with center $c = 0$, and state the radius of convergence.

(c) Approximate the value of $\int_0^1 f(x)dx$ using the Midpoint Rule for $N = 3$. You may leave your answer unsimplified.

6. The parts of this problem are NOT related.

(a) Determine the interval of convergence for which the series

$$\sum_{n=1}^{\infty} \frac{5^n (x-2)^n}{n},$$

clearly explaining and justifying your answer. Be sure to check the limiting cases.

(b) Does the series $\sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right)^{-n^2}$ conditionally? Does it converge absolutely?

(c) Calculate the Taylor series of $f(x) = \frac{1}{x^2}$ centered about $c = 4$ *as well as* identifying the interval on which the series and the function agree.

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