Problems from Math 135 Recitation on 5/1/15

These problems cover material from chapters 4 and 5. Harder problems are marked with *. For solutions see
`http://math.rutgers.edu/~az202/teaching/`.

Curve sketching:

1. Find the absolute maximum and minimum of $f(x) = 2x + 2/x$ on the interval $[1/2, 2]$.

2. Find the local extrema, intervals of increase and decrease, and asymptotes of

$$f(x) = \frac{1}{x^2 - 1}.$$

   Then sketch the graph.

3. Find the intervals where

$$f(x) = \frac{3}{3 + x^2}$$

   is concave up and concave down, and find the inflection points.

Finding limits using L'Hôpital's Rule:

4.
$$\lim_{x \to 0} \frac{x}{e^x - 1}$$

5.
$$\lim_{x \to 0} \frac{\tan(3x)}{\tan(4x)}$$

6.
$$\lim_{x \to \infty} e^{-x} \ln x$$

7. *
$$\lim_{x \to \infty} \frac{1}{\sin(1/x)} - x.$$

Optimization:

8. Show that the rectangle of area 1 that has minimum perimeter is a square.

9. A worm farmer plans to sell her friend some worms. If she prices her worms at 10 cents each, then her friend will buy 1000 worms. For every cent she adds to the price, her friend will buy 10 less worms. Assuming that the farmer has an unlimited supply of worms, and that she can sell non-integer amounts of worms, what per-unit price will maximize her profit?

10. * (Calculator needed) Bob just turned 20, and he wishes to purchase a Porsche 911 by the time he is 30. Right now, it costs \$80k, but its price increases linearly by \$3k/year. Currently, Bob only has \$70k in the bank, but it collects interest at a continuous yearly rate of 4%. Does Bob have enough savings to purchase the Porsche sometime within the next 10 years? If not, when during this timespan will he be closest to having enough, and when will he be furthest from having enough?

Integrals and the FTC:

11. Find the derivatives of the following functions:

    (a)
    $$F(x) = \int_0^x \cos(\ln t)\, dt,$$

    (b)
    $$F(x) = \int_0^{x^2} e^t \sin t\, dt.$$

12. * Show that $F(x)$, defined below, has a local maximum at $x = \sqrt{\pi}$.
    $$F(x) = \int_0^x \sin(t^2)\, dt.$$