Problems from Math 135 Recitation on 4/25/15

These problems cover material from chapters 2 and 3. Harder problems are marked with *. For solutions see
`http://math.rutgers.edu/~az202/teaching/`.

Finding limits without L'Hôpital:

1. $$\lim_{x\to 0} \frac{x^2}{e^{3x} - 3x}$$

2. $$\lim_{x\to 2} \frac{x-2}{x^2 + x - 6}$$

3. $$\lim_{x\to -\infty} \frac{5x^{10} + 6x + 1}{9x^{10} - 42x}$$

4. * $$\lim_{x\to 2^+} \frac{\sqrt{x-1} - 1}{x^2 - x - 2}$$

5. * $$\lim_{x\to \infty} e^{-x} \cos x$$

Continuity:

6. * Find A and B to make the following function continuous everywhere:

$$f(x) = \begin{cases} x + A & x < 1 \\ 3 & x = 1 \\ e^{Bx} & x > 1 \end{cases}.$$

Finding y' using differentiation rules:

7. $$y = \frac{x+1}{\sin x}$$

8. $$y = (x^3 + x)^{10}$$

9. $$y = 1/x + 1/\sqrt{x}$$

10. $$y = \ln x^8$$

11. $$y = \cos^3 x \sin(x^5)$$

12. $$y = e^{x^2}\sqrt{1+x^2}$$

13. * $$y = (\sin x)^{\tan x}.$$

Implicit differentiation:

14. Find the tangent line at $(1, 2)$ to
$$x^3 + xy + y^2 = 7.$$

15. Find y':
$$x^3 + 3x^2y - 4y^2 = 16.$$

16. * Find y':
$$e^{\sin^2 y}(1 + e^{\cos^2 y}) = xy.$$

Related rates:

17. A rectangle has dimensions x and y, diagonal z, and area A. The dimensions are changing with rates $dx/dt = 2$ and $dy/dt = 3$. At the instant $x = 4$ and $y = 3$ what are (a) dA/dt and (b) dz/dt?

18. * The volume of a sphere increases at 4π in^3/sec. At what rate is the surface area changing when the radius is 1 inch?

Linear approximation:

19. Find the linearization of $f(x) = e^x$ about $x = 0$. Use this to approximate $e^{.1}$.

20. Find the linearization of $f(x) = \sqrt{x}$ about $x = 4$. Use this to approximate $\sqrt{4.2}$.

21. * Use linearization to approximate $(\ln 5 + \ln 11 - \ln 50)\sin(1.1\pi)$. Hint: use the formula $f(x) \approx f(a) + f'(a)(x - a)$, with the appropriate choices of f, a, and x.