# Oral qualifying exam syllabus

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# 1 Primary topic: experimental mathematics

## 1.1 Numerics

- Maple coding, recursion

- Newton's method

- Ansaetze

- Lagrange interpolation, least squares, cubic splines

- Numerical integration: Newton-Cotes quadrature, Gaussian quadrature

- Iterative maps: fixed points and stability, logistic map

- ODEs: phase plane analysis, fixed points and stability, Picard-Lindelöf theorem, Euler's method (implicit and explicit), Runge-Kutta methods

- PDEs: Heat equation, Poisson equation, Laplace's equation, finite difference methods

## 1.2 Probability

- Basic definitions, conditional probability, Bayes' theorem

- Random variables: moments, conditioning, asymptotic normality, central limit theorem

- Binomial, Poisson, and geometric distributions

- Probability generating functions

- Stochastic processes: Poisson process, Markov chains, random walks, Brownian motion

- Mathematical finance: option pricing, arbitrage, Black-Scholes formula

- Simulation: Monty Hall problem, Birthday problem, Gambler's ruin

# 2 Secondary topic: combinatorics and graph theory

## 2.1 Combinatorics

- Enumeration: bijections, multinomial theorem, multisets, inclusion-exclusion, generating functions, recurrence relations, Stirling's formula

- Linear programming: weak duality, strong duality, (fractional) covering and packing

- Posets and lattices: Hasse diagrams, chains and antichains, Dilworth's theorem, distributive lattices, Birkhoff representation theorem, Möbius inversion

- Probabilistic methods: Linearity of expectation, Markov's inequality, Chebyshev's inequality, Lovász local lemma, method of alterations

- Hypergraphs: Sperner's theorem, LYM inequality, Erdős-Ko-Rado theorem, statement of Kruskal-Katona

## 2.2 Graph theory

- Basics: trees, bipartite graphs, Hamiltonian cycles and paths

- Planar graphs: Euler's formula, five color theorem, proof that $K_5$ and $K_{3,3}$ are nonplanar

- Matchings: König's theorem, Hall's theorem, Tutte's theorem

- Extremal: Mantel's theorem, Turán's theorem, Erdős-Stone theorem

- Ramsey theory: Ramsey's theorem, upper and lower bounds