# FORMULA SHEET FOR FINAL EXAM 

## Sensitivity Analysis

Assume that the optimal solution to a linear problem is represented by the final tableau

| $R$ | $\widetilde{x}_{B}$ |
| :---: | :---: |
| $\sigma-c^{T}$ | $z(\widetilde{x})$ |

where $R=B^{-1} A, \sigma=c_{B}^{T} B^{-1} A$, and $\widetilde{x}_{B}=B^{-1} b$.
Changes to one coefficient of the objective function:
If $x_{\ell}$ is not a basic variable, then $\Delta c_{\ell} \leq \sigma_{\ell}-c_{\ell}$.
If $x_{\ell}$ is the basic variable of row $i$, then $-\Delta c_{\ell} R_{i j} \leq \sigma_{j}-c_{j}$.

## Changes to one entry of the resource vector:

$\widetilde{x}_{B}+\Delta b_{\ell} B^{-1} e_{\ell} \geq 0$.

## Cutting Plane for Mixed Integer Programs

Assume that $x=\left(x_{1}, x_{2}, \ldots, x_{s}\right) \in \mathbb{R}^{s}$ satisfies

- $a_{1} x_{1}+a_{2} x_{2}+\cdots+a_{s} x_{s}=b \quad$ where $a_{j}, b \in \mathbb{R}$
- $x_{j} \geq 0$ for $1 \leq j \leq s$
- $x_{j} \in \mathbb{Z}$ for $j \in I \quad$ where $I \subset\{1,2, \ldots, s\}$ is a given subset.

Set $g_{j}=\operatorname{frac}\left(a_{j}\right)$ and $f=\operatorname{frac}(b)$ and
$d_{j}= \begin{cases}g_{j} & \text { if } j \in I \text { and } g_{j} \leq f \\ \frac{f}{1-f}\left(1-g_{j}\right) & \text { if } j \in I \text { and } g_{j}>f \\ a_{j} & \text { if } j \notin I \text { and } a_{j} \geq 0 \\ \frac{f}{1-f}\left(-a_{j}\right) & \text { if } j \notin I \text { and } a_{j}<0 .\end{cases}$
Then $d_{1} x_{1}+d_{2} x_{2}+\cdots+d_{s} x_{s} \geq f$.

