Various Fourier-type expansions:

We write $f(x) \sim$ series to indicate that the given series is some Fourier-type expansion of $f(x)$. All the formulas for coefficients come from the formula

$$c_n = \frac{(f, \varphi_n)}{(\varphi_n, \varphi_n)}$$

with $\varphi_1, \varphi_2, \dots$ a complete orthogonal set on $[-p, p]$ or $[0, L]$.

$$\left.\begin{aligned} f(x) &\sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\frac{n\pi x}{p} + b_n \sin\frac{n\pi x}{p} \right]; \\ a_0 = \frac{1}{p}\int_{-p}^{p} f(x)\,dx, \quad a_n &= \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi x}{p}\,dx, \quad b_n = \frac{1}{p}\int_{-p}^{p} f(x)\sin\frac{n\pi x}{p}\,dx. \end{aligned}\right\} \qquad \textbf{(1)}$$

$$f(x) \sim \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/p}; \qquad c_n = \frac{1}{2p}\int_{-p}^{p} f(x) e^{-in\pi x/p}\,dx. \qquad \textbf{(2)}$$

$$\left.\begin{aligned} f(x) &\sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}; \\ a_0 = \frac{2}{L}\int_0^L f(x)\,dx, \quad a_n &= \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx. \end{aligned}\right\} \qquad \textbf{(3)}$$

$$f(x) \sim \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}; \qquad b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx. \qquad \textbf{(4)}$$

Some antiderivatives:

$$\int e^{-ax}\sin(bx)\,dx = -\frac{e^{-ax}}{a^2+b^2}\left(a\sin(bx) + b\cos(bx)\right) + C$$

$$\int e^{-ax}\cos(bx)\,dx = \frac{e^{-ax}}{a^2+b^2}\left(-a\cos(bx) + b\sin(bx)\right) + C$$

$$\int x\cos(bx)\,dx = \frac{\cos(bx)}{b^2} + \frac{x\sin(bx)}{b} + C \qquad \int x\sin(bx)\,dx = \frac{\sin(bx)}{b^2} - \frac{x\cos(bx)}{b} + C$$

Some trig identities:

$$\cos^2 x = \frac{1+\cos(2x)}{2} \qquad \sin^2 x = \frac{1-\cos(2x)}{2}$$

$$\cos(A)\cos(B) = (1/2)\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin(A)\cos(B) = (1/2)\left[\sin(A+B) + \sin(A-B)\right]$$

$$\sin(A)\sin(B) = (1/2)\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B, \qquad \cos(A+B) = \cos A\cos B - \sin A\sin B$$

Hyperbolic functions:

$$\cosh'(x) = \sinh x, \qquad \sinh'(x) = \cosh x$$

Sturm-Liouville problem:

$$[r(x)y']' + (q(x) + \lambda p(x))y = 0, \qquad (f, g)_p = \int_0^L f(x)g(x)p(x)\,dx$$