Problem 1. Let $X$ be an affine variety such that the affine coordinate ring $A(X)$ is a unique factorization domain. Let $U \subset X$ be an open subset. Show that if $f: U \to k$ is any regular function, then there exist $p, q \in A(X)$ such that $q(x) \neq 0$ and $f(x) = p(x)/q(x)$ for all $x \in U$.

Problem 2. (a) $k[A^n \setminus \{0\}] = k[x_1, \ldots, x_n]$ for $n \geq 2$.
   (b) $A^n \setminus \{0\}$ is not an affine variety for $n \geq 2$.
   (c) Every global regular function on $\mathbb{P}^n$ is constant, i.e. $k[\mathbb{P}^n] = k$.
   (d) $\mathbb{P}^n$ is not quasi-affine for $n \geq 1$.

Problem 3. Let $\phi: A^1 \to V(y^2 - x^3) \subset A^2$ be the morphism given by $\phi(t) = (t^2, t^3)$. Show that $\phi$ is bijective, but not an isomorphism.

Problem 4. Let $X \subset A^n$ be a closed subvariety. Identify $A^n$ with $D_+^{x_0} \subset \mathbb{P}^n$ and let $X$ be the closure of $X$ in $\mathbb{P}^n$. Show that $I(\overline{X}) = I(X)^* \subset k[x_0, \ldots, x_n]$. ($I(X)^*$ was defined in class on 9/20.)

Problem 5. Let $X \subset \mathbb{P}^n$ be a projective variety with projective coordinate ring $R = k[x_0, \ldots, x_n]/I(X)$. Let $f \in R$ be a non-constant homogeneous element. Show that $D_+(f) \subset X$ is an open affine subvariety with affine coordinate ring $k[D_+(f)] = R(f)$.

Problem 6. Show that if $R$ is a finitely generated reduced $k$-algebra then the space with functions Spec-$m(R)$ is an affine variety.

Problem 7. Let $X$ be any space with functions. A map $\phi: \mathbb{P}^n \to X$ is a morphism if and only if $\phi \circ \pi: A^{n+1} \setminus \{0\} \to X$ is a morphism.

Problem 8. Let $X$ and $Y$ be spaces with functions and let $(P, \pi_X, \pi_Y)$ and $(P', \pi'_X, \pi'_Y)$ be two products of $X$ and $Y$. Show that there is a unique isomorphism $\phi: P \iso P'$ such that $\pi_X = \pi'_X \circ \phi$ and $\pi_Y = \pi'_Y \circ \phi$. 