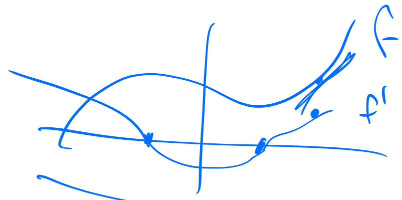
Last time: Derivative of a function $y=f(x)$ is a new function $f'(x)$ $= \frac{df}{dx} = \frac{dy}{dx}$, which measures slope of tangent line at pt (x,y)



$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{u\to x} \frac{f(u)-f(x)}{u-x}$$

$$\frac{d}{dx}(x^2) = 2x, \quad \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2},$$

$$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}, \quad \frac{d}{dx}\left(x^2+\frac{1}{x}\right) = 2x - \frac{1}{x^2}.$$

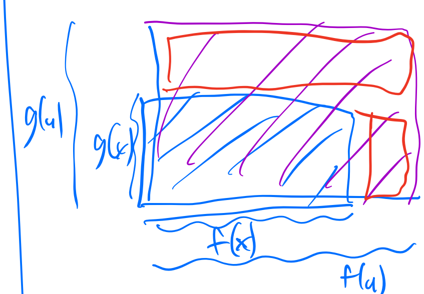
"Sum Rule" $(f+g)' = f'+g'$

"Constant Rule": $\frac{d}{dx}(c) = 0.$

$$(f\cdot g)' \cancel{=} f'\cdot g'$$

$$\frac{d}{dx}(x\cdot x) \cancel{=} \frac{d}{dx}(x)\cdot\frac{d}{dx}(x) = 1$$

$$(f\cdot g)'(x) = \lim_{u\to x} \frac{f(u)\cdot g(u) - f(x)\cdot g(x)}{u-x}$$



$$f(u)\cdot g(u) - f(x)\cdot g(x)$$
$$= (f(u)-f(x))\cdot g(x)$$
$$+ f(u)\cdot(g(u)-g(x)).$$

$$\lim_{u\to x} \frac{(f(u)-f(x))}{u-x}\cdot g(x) + f(u)\frac{(g(u)-g(x))}{u-x}$$

$$= g(x)\cdot f'(x) + f(x)\cdot g'(x).$$

$\boxed{\text{Product Rule}}$

$$(f\cdot g)' = f'\cdot g + f\cdot g'.$$

$$\frac{d}{dx}[x\cdot x] = \frac{d}{dx}[x]\cdot x + x\cdot\frac{d}{dx}[x] = 2x$$

$$\frac{d}{dx}[x^2\cdot x] = 2x\cdot x + x^2\cdot 1$$
$$= 3x^2$$

Quiz: $$\frac{d}{dx}[x^3\cdot x] = 3x^2\cdot x + x^3\cdot 1$$
$$= 4x^3.$$

Will's solution $$\frac{d}{dx}(x^2\cdot x^2) = 2x\cdot x^2 + x^2\cdot 2x = 4x^3.$$

Power Rule:

$$\frac{d(x^\alpha)}{dx} = \alpha\cdot x^{\alpha-1}.$$

Recap:

$$\frac{d(x^{-1})}{dx} = -1\cdot x^{-2}$$
$$\frac{d(x^0)}{dx} = 0\cdot x^{-1}$$
$$\frac{d(x)}{dx} = 1 = 1\cdot x^0 \quad \frac{d(x^{1/2})}{dx} = \frac{1}{2}x^{-1/2}$$
$$\frac{d(x^2)}{dx} = 2x$$
$$\frac{d(x^3)}{dx} = 3x^2$$
$$\frac{d(x^4)}{dx} = 4x^3$$

$$u^5 - x^5 = u\left(u^4 + \cancel{u^3\cdot x} + \cancel{u^2 x^2} + \cancel{\dots} + x^4\right)$$
$$-x\left(\cancel{u^4} + \cancel{u^3\cdot x} + \cancel{u^2x^2} + \dots + x^4\right)$$

$$\frac{d(x^5)}{dx} = \lim_{u\to x}\frac{u^5-x^5}{u-x} = \lim_{u\to x}\left(u^4 + u^3 x + u^2x^2 + u x^3 + x^4\right)$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$x^4 \quad x^4 \quad x^4 \quad x^4 \quad x^4$$
$$= 5x^4.$$

$5 \to n$

Reciprocal Rule:

$$\frac{d}{dx}\left[\frac{1}{f(x)}\right] = \lim_{u\to x}\frac{\frac{1}{f(u)} - \frac{1}{f(x)}}{u-x}$$

$$= \lim_{u\to x}\frac{f(x)-f(u)}{f(u)\cdot f(x)\,(u-x)} = \lim_{u\to x}\frac{-1}{f(u)\cdot f(x)}\cdot\left(\frac{f(u)-f(x)}{u-x}\right)$$
$$\downarrow \qquad\qquad \downarrow$$
$$\frac{-1}{f(x)^2} \cdot f'(x)$$

$$\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}.$$

$$\frac{d}{dx}\left[\overset{\frac{1}{x^n}}{x^{-n}}\right] = \frac{-(n\cdot x^{n-1})}{x^{2n}}$$

$$= (-n)\cdot x^{-n-1}$$

Rule Roundup:

Power Rule: $\frac{d}{dx}\{x^{\alpha}\} = \alpha\cdot x^{\alpha-1}$

Product Rule: $(f\cdot g)' = f'\cdot g + g'\cdot f.$

Reciprocal Rule: $\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}.$

$\boxed{\text{Quotient Rule:}}$ $\left(\frac{f}{g}\right)' = ?$

$\boxed{\text{Best Song}}$

Low d(High) − High d(Low)

over the square of what's below.

$$\left[f\cdot\frac{1}{g}\right]' = f'\cdot\frac{1}{g} + f\cdot\left[\frac{1}{g}\right]'$$

$$= \frac{f'}{g} + f\cdot\left[\frac{-g'}{g^2}\right]$$

$$= \boxed{\frac{f'\cdot g - f\cdot g'}{g^2} = \left[\frac{f}{g}\right]'}$$

$$\frac{d}{dx}\left(\frac{x}{x-1}\right) = \frac{(x-1)\cdot 1 - x\cdot 1}{(x-1)^2}$$

$$= \frac{-1}{(x-1)^2}.$$

$$(c\cdot f)' = \cancel{c'\cdot f} + c\cdot f'$$

Quiz: $\frac{d}{dx}\left[\frac{4x^2+x-7}{x^3+13}\right]$.

$$=\frac{(x^3+13)(4\cdot 2\cdot x^1+1+0)-(4x^2+x-7)(3x^2+0)}{\left(x^3+13\right)^2}$$

$(x-1)^2=x^2-2x+1$

$\hookrightarrow \frac{d}{dx}=2x-2$

Repeated differentiation:

f', $f''=\frac{\partial^2}{dx^2}(f)=\frac{d^2y}{dx^2}=D^2f$

second derivative

"f double-prime"

E.g.: $\left[\frac{x}{x-1}\right]''=\left[\frac{-1}{(x-1)^2}\right]'$

$=-1\left[\frac{-(2x-2)}{(x-1)^4}\right]$

Reciprocal Rule

$=\frac{2}{(x-1)^3}$.