Last time: $\frac{d}{dx}e^x = e^x$, $\frac{d}{dx}(x^\alpha) = \alpha x^{\alpha-1}$ ✓

$\frac{d}{dx}[\sin x] = \cos x$ ✓, $(f\cdot g)' = f'\cdot g + f\cdot g'$ ✓, $\left(\frac{f}{g}\right)' = \frac{g\cdot f' - f\cdot g'}{g^2}$.

$\frac{d}{dx}(f(g(x))) = f'(g(x))\cdot g'(x)$. $\Rightarrow$ Implicit Differentiation.

$$\frac{d}{dx}(\ln x) = \lim_{y\to x}\frac{\ln y - \ln x}{y-x} \Rightarrow \ln\frac{y}{x} \ \ldots ??,\ \to \frac{1}{x}.$$

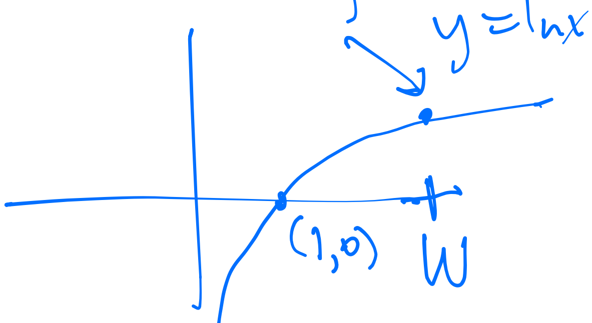
Do know $\frac{d}{dx}[e^x] = e^x$ ... ?? If $y = \ln x$, then $e^y = e^{\ln x}$

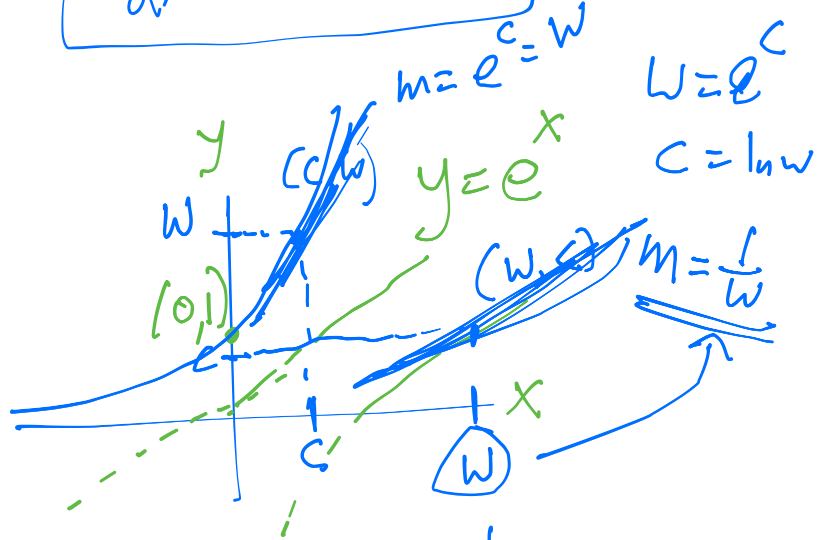
$X = e^y$ Implicit Diffi $1 = \frac{d}{dx}(X) = \frac{d}{dx}(e^y) = e^y \cdot y'$.

$y' = \frac{1}{e^y} = \frac{1}{x}$.

$$\boxed{\frac{d}{dx}\ln x = \frac{1}{x}}$$

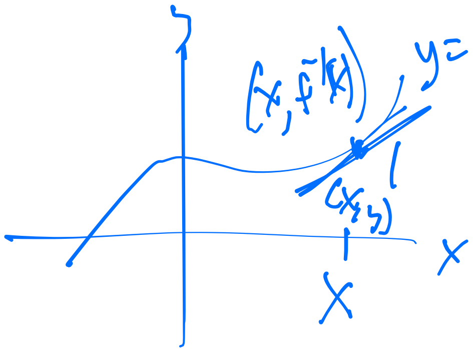
Geometrically:

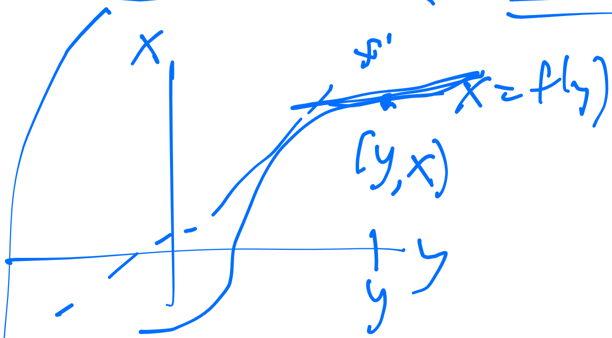




Inverse function $x \leftrightarrow y$, rise $\leftrightarrow$ run, $m \leftrightarrow \frac{1}{m}$.

More generally: If $\underbrace{y = f^{-1}(x)}$ $\Leftrightarrow$ $\underline{x = f(y)}$





Diff both: $1 = \frac{d}{dx} x = \frac{d}{dx}[f(y)] = f'(y) \cdot y'$.

$$y' = \frac{1}{f'(y)} = \frac{1}{f'(f^{-1}(x))}$$

$$\boxed{\text{Inverse Rule: } (f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}}$$

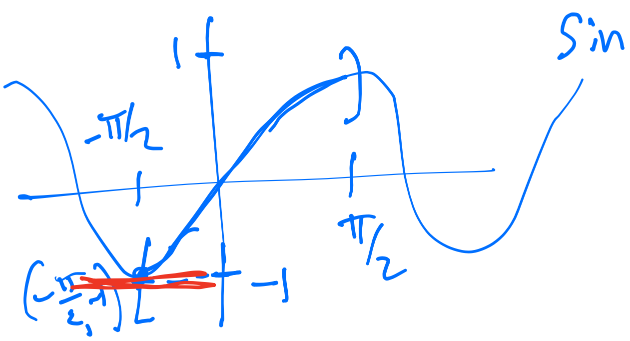
Try $f(x) = e^x$. $f'(x) = e^x$, $f^{-1}(x) = \ln x$.

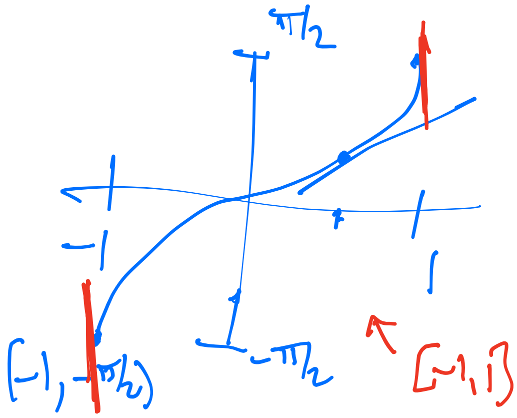
$$(\ln x)' = \frac{1}{e^{(\ln x)}} = \frac{1}{x}$$

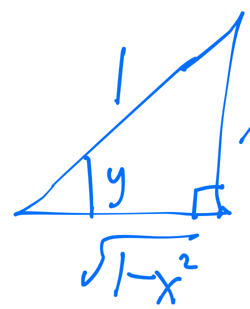
$$\frac{d}{dx}\left[\sin^{-1}(x)\right] = ??$$

$\underbrace{y = \sin^{-1} x}$

$y = \sin x$







$$\boxed{x = \sin y} \Rightarrow 1 = \cos y \cdot y'.$$

$$y' = \frac{1}{\cos y} = \frac{1}{\cos(\sin^{-1}x)} = \frac{1}{\sqrt{1-x^2}} \quad (|x|<1).$$

$$\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}.$$

Inverse Rule:

$$\left(f^{-1}(x)\right)' = \frac{1}{f'(f^{-1}(x))}.$$

$$\frac{d}{dx}\left(\cos^{-1}x\right) = \frac{1}{-\sin y}$$

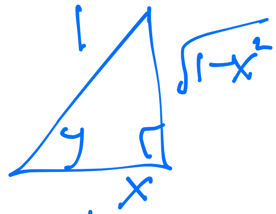
$$= -\frac{1}{\sin(\cos^{-1}x)}$$

$$= -\frac{1}{\sqrt{1-x^2}}.$$

$y = \cos^{-1}x$

$x = \cos y$

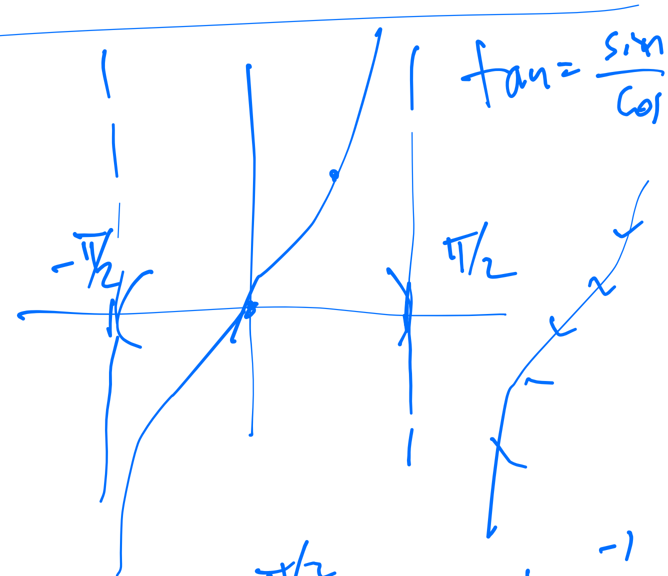
$1 = -\sin y \cdot y'$



$$\boxed{\frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2}}$$

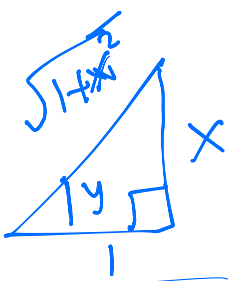
$$= \frac{1}{\sec^2(\tan^{-1}x)} = \frac{1}{\left(\sqrt{1+x^2}\right)^2}$$

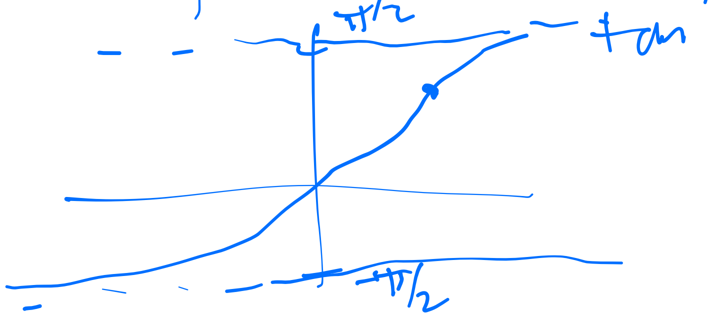
$\tan = \frac{\sin}{\cos}$



$y = \tan^{-1}x$

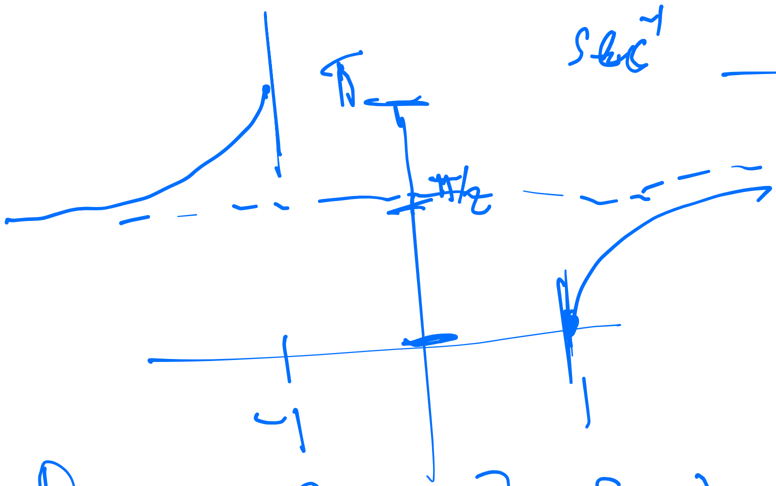
$x = \tan y$, $\frac{x}{1}$

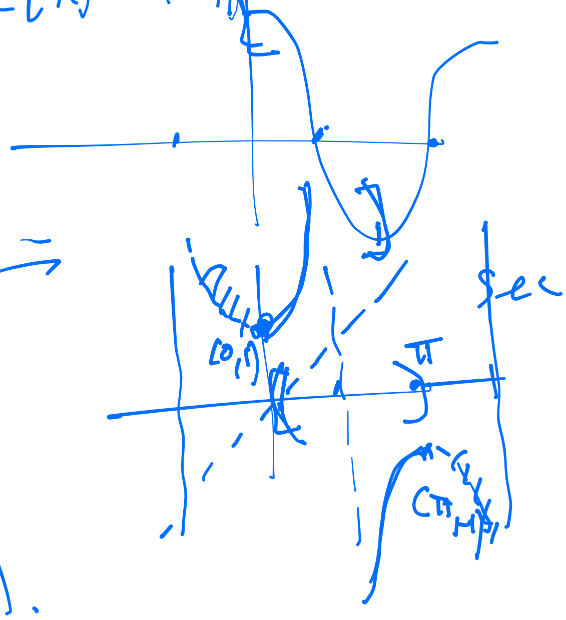




$$\frac{d}{dx}\sec x = \sec x \tan x$$

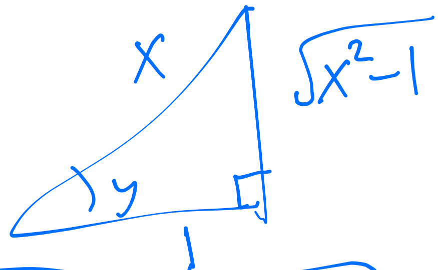
Quiz: $\frac{d}{dx}\left(\sec^{-1}x\right) = \frac{1}{\sec(\sec^{-1}x)\tan(\sec^{-1}x)}$ (a)





Domain: $(-\infty,-1] \cup [1,\infty)$.

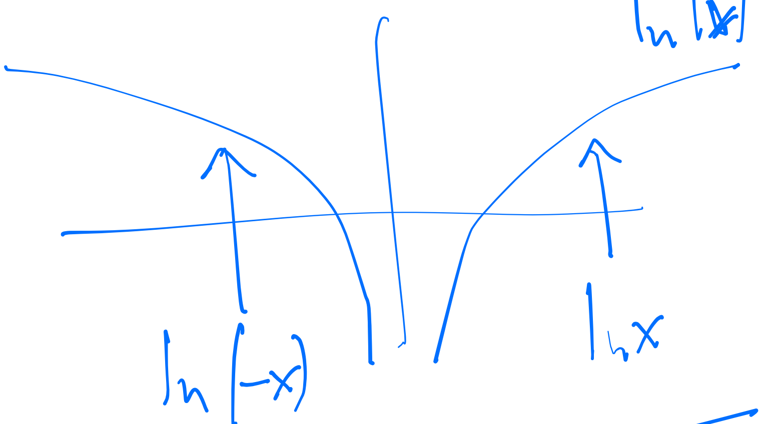
$y = \sec^{-1}x$, $\boxed{x = \sec y}$



$1 = \sec y \cdot \tan y \cdot y'$

$= x \cdot \sqrt{x^2-1} \cdot y'$, $\boxed{y' = \frac{1}{|x|\sqrt{x^2-1}}}$ $(x>0)$ $(x<0)$

$$\frac{d}{dx}\left(\ln|x|\right) = \begin{cases} \frac{1}{x}, & x>0 \\ \frac{1}{x}, & x<0 \end{cases}$$

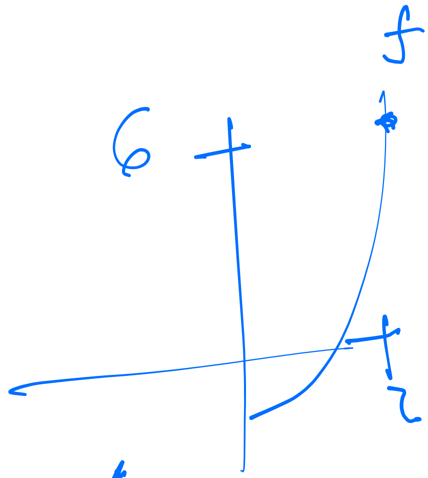


$x<0$: $\frac{d}{dx}\left(\ln(-x)\right) = \frac{1}{-x}\cdot(-1)$ $\boxed{\frac{d}{dx}\ln|x| = \frac{1}{x}}$

Eg: $\frac{d}{dx} \ln(x^2+3) = \frac{1}{x^2+3} \cdot (2x).$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$$

Eg: $f(x) = x^3 - 2, \quad x \geq 0$



Q: $(f^{-1})'(6) = \frac{1}{3(2)^2}$

$f'(x) = 3x^2$

$f^{-1}(6) = ? = 2$

$x^3 - 2 = 6, \quad x^3 = 8, \quad x = 2$

Exercise: Compute $f^{-1} = \sqrt[3]{x+2}$.

$(f^{-1})'(6)$