Recall: F-distribution: $\boxed{F = \dfrac{U/n}{V/m}}$

$U \sim \chi^2$ with n deg freedom
$V \sim \chi^2$ ... m ...
$\Big\}$ independent.

$$P(F \le a) = \int_{v=0}^{\infty} \int_{u=0}^{a v \frac{n}{m}} f_{\chi^2_n}(u)\, du \; f_{\chi^2_m}(v)\, dv$$

$$\frac{u/n}{v/m} \le a$$

Let $t = \dfrac{u/n}{v/m}$, $\quad \dfrac{dt}{t} = \dfrac{du}{u}$

$$u = t\, v \frac{n}{m}$$

$$= \int_{v=0}^{\infty} \int_{t=0}^{a} \frac{1}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} e^{-(t v \frac{n}{m})/2} (t)^{n/2} \left(v \frac{n}{m}\right)^{n/2} \frac{dt}{t} f_{\chi^2_m}(v)\, dv$$

$$= \int_{t=0}^{a} \frac{\left(\frac{n}{m}\right)^{n/2}}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} \int_{v=0}^{\infty} e^{-(t v \frac{n}{m})/2}\, v^{n/2} \frac{1}{2^{\frac{m}{2}}\Gamma\left(\frac{m}{2}\right)} e^{-v/2}\, v^{m/2} \frac{dv}{v}\, t^{n/2} \frac{dt}{t}$$

$$\boxed{v = \frac{z \cdot 2}{\left(1 + t\frac{n}{m}\right)}} \qquad e^{-v\left(1 + t\frac{n}{m}\right)/2} \qquad \boxed{v^{(n+m)/2}}$$

Let $z = v\left(1 + t\frac{n}{m}\right)/2$ $\qquad \dfrac{dv}{v} = \dfrac{dz}{z}$

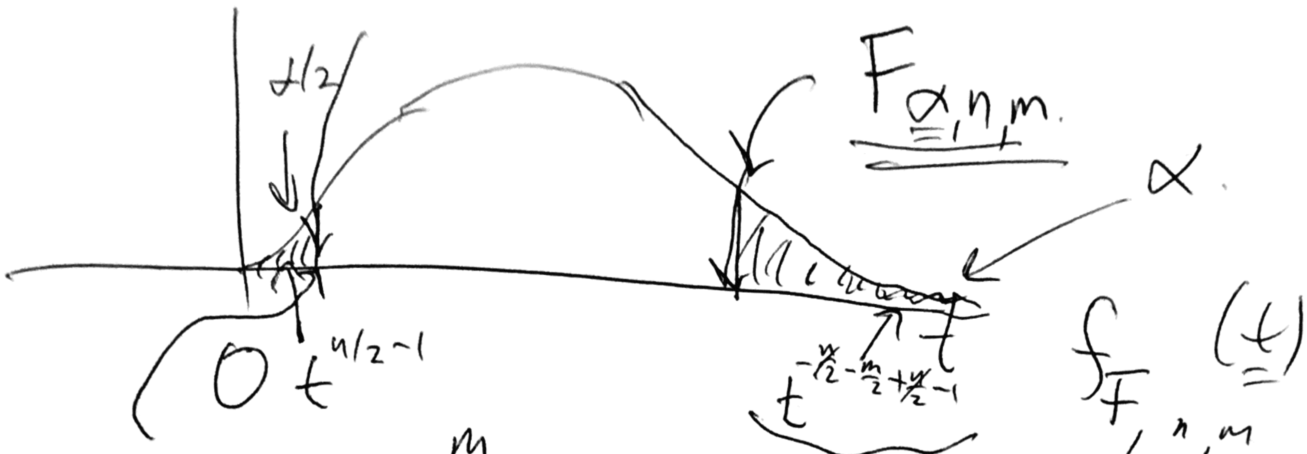
$$= \int_{t=0}^{a} \frac{\left(\frac{n}{m}\right)^{n/2}}{2^{n/2}\Gamma\left(\frac{n}{2}\right) 2^{m/2}\Gamma\left(\frac{m}{2}\right)} \int_{z=0}^{\infty} e^{-z} \left[\frac{z \cdot 2}{\left(1+t\frac{n}{m}\right)}\right]^{\frac{n+m}{2}} \frac{dz}{z}\, t^{n/2} \frac{dt}{t}$$

$$= \int_{t=0}^{a} \underbrace{\frac{\left(\frac{n}{m}\right)^{n/2}}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(\frac{m}{2}\right)} \Gamma\left(\frac{n+m}{2}\right) \cdot \left(1+t\frac{n}{m}\right)^{-\frac{(n+m)}{2}} t^{n/2} \frac{1}{t}}_{f_F(t) = \text{" } f_{F,n,m}(t)} dt$$

S_A^2, S_B^2, $X_1^{(A)}, \dots, X_{n_A}^{(A)}$, $X_1^{(B)}, \dots, X_{n_B}^{(B)}$ iid Gaussian

$$U = \frac{(n_A-1) S_A^2}{\sigma_A^2}, \quad S_A^2 = \frac{1}{(n_A-1)} \sum_{i=1}^{n_A} \left(X_i^{(A)} - \overline{X}^{(A)}\right)^2$$

$$F = \frac{U/n}{V/m} = \frac{\frac{(n_A-1) S_A^2}{\sigma_A^2} \Big/ (n_A-1)}{\frac{(n_B-1) S_B^2}{\sigma_B^2} \Big/ (n_B-1)} = \frac{S_A^2 \sigma_B^2}{S_B^2 \sigma_A^2}$$



$F_{1-\frac{\alpha}{2},} \qquad t^{-\frac{m}{2}-1} = t^{-\frac{m}{2}-1} \qquad (n,m) \geq 3.$

So ~~a~~ $(1-\alpha)100\%$ confidence interval for $\dfrac{\sigma_A^2}{\sigma_B^2}$ is given by:

$$\mathbb{P}\Bigg(\underbrace{F_{1-\frac{\alpha}{2}, n_A-1, n_B-1} \leq \frac{S_A^2\,\sigma_B^2}{S_B^2\,\sigma_A^2} \leq F_{\frac{\alpha}{2}, n_A-1, n_B-1}}\Bigg) = 1-\alpha.$$

$$= \mathbb{P}\left(\frac{1}{F_{\frac{\alpha}{2}, n_A-1, n_B-1}} \cdot \frac{S_A^2}{S_B^2} \leq \frac{\sigma_A^2}{\sigma_B^2} \leq \frac{S_A^2}{S_B^2} \cdot \frac{1}{F_{1-\frac{\alpha}{2}, n_A-1, n_B-1}}\right)$$