Math 477, Summer 2018

Homework 3

Due June 7th

Name:

Show work and include justification when necessary. For part 1 short justifications can be used. For part 2 the justifications and proofs must be written using complete sentences. Part 2 will be graded both on mathematical correctness and quality of writing. Remember a longer proof is not often better- try to be concise and precise with your writing.
Part 1 and 2 should be turned in separately.
I use the notation $\mathbb{P}(A)$ for the probability of the event A.

Part 1

(1) Three distinct numbers from $\{1, 2, ..., 12\}$ were chosen. If the largest was 8, what is the probability that the number 4 is among the chosen ones?

(2) Three cards were drawn (one-by-one)from a regular 52-card deck.
   (a) If the first card was a heart, what is the probability that all three cards are hearts?
   (b) If at least one of the cards was of hearts, what is the probability that all three cards are hearts

(3) A new light bulb has a 15% chance of lasting up to a month, a 20% chance of lasting more than a month but less than a year, and a 65% chance of lasting more than a year. If my bulb is still on after 1 month since I bought it, what is the probability that it will last more than a year?

(4) If eight defective and twelve nondefective items are inspected one-by-one, at random and without replacement, what is the probability that
   (a) the first four items inspected are defective?
   (b) from the first three items at least two are defective?

(5) Sam is known to speak truthfully 80% of the time. If he rolls a standard die and reports that it landed on a 3 what is the probability the die actually landed on a 3? (If Sam lies he picks one of the remaining 5 options uniformly.)

(6) Suppose events A, B, C are independent, and $\mathbb{P}(A) = 1/2$, $\mathbb{P}(B) = 1/5$, $\mathbb{P}(C) = 1/4$. What is the probability of exactly one of them happening?

(7) An urn has 50 normal dice, 20 dice whose face numbers are $2, 2, 4, 4, 6, 6$, and 20 dice whose face numbers are $1, 1, 3, 3, 5, 5$. One die was chosen at random.

(a) What is the probability of this die rolling a 6?

(b) If I just rolled one 6 with this die, what is the probability that I get a 6 on my next roll of it?

(c) If I just rolled two 6's with this die, what is the probability that I get a 6 on my next roll of it?

Hint: Hint: Instead of defining events

E1 = Rolled six on first throw

E2 = Rolled six on second throw

E3 = Rolled six on third throw

consider defining the events

E1 = Rolled six on first throw

E2 = Rolled six on first and second throws

E3 = Rolled six on first, second and third throws

Part 2

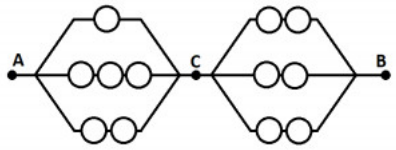
(1) The World Series of baseball is a series of at most 7 games played between two teams, A and B. This series is played only until one of the teams has accumulated 4 total wins, in which case it is declared the champion. Therefore sometimes all 7 games are needed, but not always. Suppose team A has a probability p of winning each game, and B has $1 - p$, independently of other games (no ties are possible in baseball).

What is the probability, in terms of p, that 7 games will be needed?

Sanity check: Your answer should clearly be 0 when plugging-in $p = 0$ or $p = 1$ (why?). Is it?

(2) Each circle is a door that has chance p of being open, independently of the other doors. What is the probability that there is an open path from A to B?

Hint: Find first the probability of an open path from A to C, then the probability of an open path from C to B. What do you do with these probabilities?



(3) Let A, B, C, D be independent events. Prove that

$$\mathbb{P}(A \cup B \cup C \cup D) = 1 - (1 - \mathbb{P}(A))(1 - \mathbb{P}(B))(1 - \mathbb{P}(C))(1 - \mathbb{P}(D))$$

either using De Morgans laws or expanding the right-hand side product and using the inclusion-exclusion principle.