Math 477 Problems on Continuous Random Variables

1. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

for some value of c. What is the value of c? What is the cumulative distribution function of X?

2. The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = xe^{-x} \quad x \geq 0.$$

Compute the expected lifetime of such a tube.

3. Let X be a continuous random variable, with cumulative distribution function $F_X(x)$ and probability density function $f_X(x)$. Let $Y = 5X - 2$. Find an expression for the probability density function $f_Y(y)$ of Y.

4. The lifetime of a machine part has a continuous distribution on the interval $(0, 40)$, with probability density function

$$f(x) = \begin{cases} \dfrac{c}{(10+x)^2} & 0 < x < 40, \\ 0 & \text{otherwise,} \end{cases}$$

for some value c. What is the probability that the lifetime of the machine part is less than 5?

5. Trains headed for destination A arrive at the train station at 15-minute intervals tarting at 7am, where as trains headed for destination B arrive at 15-minute intervals tarting at 7:05am.

   (a) If a certain passenger arrives at the station at a time uniformly distributed between 7 and 8am, and then gets on the first train that arrives, what proportion of time does he or she go to destination A?

   (b) What if the passenger arrives at a time uniformly distributed between 7:10am and 8:10am?

6. You arrive at a bus stop at 10am, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

   (a) What is the probability that you will need to wait longer than 10 minutes?

   (b) If, at 10:15, the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

7. Let X be a normal random variable with mean 4 and variance 9. Compute

   (a) $P(3 < X < 6)$,

   (b) $P(X > 0)$,

   (c) $P(X < 20)$,

   (d) $P(|X - 4| > 3)$.

8. You flip a biased coin 100 times, and the probability of heads is 0.6. Approximate the probability that:

   (a) you would get strictly more than 65 heads,

   (b) you would get strictly less than 53 heads,

   (c) you would get between 54 and 63 heads inclusively.

9. The salaries of physicians are approximately normally distributed. If 25 percent of these physicians earn less than \$180,000, and 25 percent earn more than \$320,000, approximately what fraction earn less than \$200,000? Approximately what fraction earn between \$280,000 and \$320,000?

10. Let X be a normal random variable with mean 12 and variance 4. Find the value z such that $P(X > z) = 0.10$.

11. Suppose that the number of miles that a car can run before its battery dies is exponentially distributed with an average value of 5000 miles. If Jane desires to take a a 2000-mile trip, what is the probability that she will be able to complete the trip without replacing the battery?

12. The number of years a TV functions is exponentially distributed, with mean 5 years. If Smith buys a used TV, what is the probability that it will be working an additional 10 years?

13. The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following the purchase, and a one-half refund if it fails the second year.

    If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

14. The time, in hours, to repair a machine is an exponentially distributed random variable with mean 2.

    (a) What is the probability that a repair time exceeds 2 hours?

    (b) What is the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours?