Math 477 Problems for Discrete Random Variables

1. Two fair dice are rolled. Let X be the product of the two numbers that are rolled.

    (a) Compute $\mathbb{P}(X = i)$, for $1 \leq i \leq 5$.
    (b) Compute $\mathbb{E}[X]$.

2. Let X be the winnings of a gambler and assume that

$$\mathbb{P}(X = 0) = \frac{1}{3}, \quad \mathbb{P}(X = 1) = \frac{13}{55}, \quad \mathbb{P}(X = -1) = \frac{13}{55},$$

$$\mathbb{P}(X = 2) = \frac{1}{11}, \quad \mathbb{P}(X = -2) = \frac{1}{11}, \quad \mathbb{P}(X = 3) = \frac{1}{165}, \quad \mathbb{P}(X = -3) = \frac{1}{165}.$$

    (a) Compute the conditional probability that gambler wins i, for $i = 1, 2, 3$, given that he wins a positive amount.
    (b) Compute $\mathbb{E}[X]$.
    (c) Compute $\mathrm{Var}(X)$.

3. The probability mass function of the discrete random variable X is

$$\mathbb{P}(X = 0) = 0.1, \quad \mathbb{P}(X = 1) = 0.3, \quad \mathbb{P}(X = 2) = 0.4, \quad \mathbb{P}(X = 3) = 0.2,$$

    and $\mathbb{P}(X = x) = 0$ if $x \neq 0, 1, 2, 3$.

    (a) Compute $\mathbb{E}[2X^2 - 3X + 1]$.
    (b) Compute $\mathbb{E}[\sin(X)]$.

4. The probability that you win i dollars, for $i = 1, 2, 3, 4, 5$, is proportional to $\frac{1}{i}$, that is, there is a constant c such that the probability you win i dollars is $c\frac{1}{i}$.

    (a) What is the probability of winning i dollars for $1 \leq i \leq 5$?
    (b) Let X be the amount won. What is $\mathbb{E}[X]$. What is $\mathrm{Var}(X)$?

5. The number of injury claims per month is modeled by a random variable N with

$$\mathbb{P}(N = n) = \frac{1}{(n + 1)(n + 2)}, \quad \text{where } n \geq 0.$$

    Determine the probability of at least one claim during a particular month, given that there have been at most four claims during that month.

6. A sample of 3 items is selected at random from a box containing 20 items, of which 4 are defective. Find the expected number of defective items in the sample.

7. In a multiple-choice test with 5 possible answers for each of the 10 questions, what is the probability that the student will get 3 or more correct answers just by guessing?

8. What is the probability that when you toss a fair coin n times, the number of heads is strictly larger than a and strictly less than b, if $0 \leq a < b \leq n$?

9. A certain loaded die has

$$\mathbb{P}(X = 1) = 0.1, \quad \mathbb{P}(X = 2) = 0.2, \quad \mathbb{P}(X = 3) = 0.3,$$

$$\mathbb{P}(X = 4) = 0.1, \quad \mathbb{P}(X = 5) = 0.2, \quad \mathbb{P}(X = 6) = 0.1.$$

    If you roll it 100 times, what is the expected number of times it lands on a prime number? What is the standard deviation? Note that 1 is not a prime number.

10. In a multiple choice test, there are 20 questions, and each question has four choices. Jane is clueless and answers by purely guessing. She is equally likely to pick any of the possible choices, and each such choice is independent of the other ones. If it is known that Jane guessed correctly strictly more than six questions correctly, what is the chance that she scored strictly more than eight questions correctly?

11. The probability mass function of random variable X is given by

$$\mathbb{P}(X = i) = \begin{cases} e^{-2}\dfrac{2^i}{i!} & \text{for } i = 0, 1, 2, \cdots \\ 0 & \text{otherwise.} \end{cases}$$

Verify that $\sum_{i=1}^{\infty} \mathbb{P}(X = i) = 1$. Compute $\mathbb{E}[X]$.