Math 477 Problems in Conditional Probability

1. Two fair dice are rolled. What is the conditional probability that at least one lands on 6, given that the dice land on different numbers?

2. A coin is flipped three times. Assume that all the eight outcomes

$$\{hhh, hht, hth, htt, thh, tht, tth, ttt\}$$

are equally likely. What is the conditional probability that all flips landed on head, given that

   (a) the first two flips were heads

   (b) the first and last flip were heads

   (c) at least one flip was a head

   (d) at least two flips were heads

   (e) all three flips were heads.

3. A recent college graduate is planning to take the first three actuarial exams in the coming summer. If she passes the first exam, she will be allowed to take the second exam, and if she passes the second exam, she will be allowed to take the third exam. If she fails an exam, then she is not allowed to take any other exam.

   - The probability that she passes the first exam is 0.9.
   - The probability that she passes the second exam, given that she passes the first one, is 0.8.
   - The conditional probability that she passes all three exams, given that she passed the first two exams, is 0.7.

   (a) What is the probability that she passes all three exams?

   (b) Given that she did not pass all three exams, what is the conditional probability that she failed the second exam?

4. An ectopic pregnancy is twice as likely to develop when the pregnant woman is a smoker as it is when she is a nonsmoker. If 32% of women of childbearing age are smokers, what percentage of women having ectopic pregnanices are smokers?

5. The probability that someone smokes is 0.3. The probability that a smoker would eventually die of lung cancer is 0.1. What is the probability that a random deceased person is a smoker who died of lung cancer?

6. Consider two boxes, one containing 1 black and 1 white marble, the other containing 2 black and 1 white marble. A box is selected at random, and a marble is drawn from it at random. What is the probability that the marble is black? What is the probability that the first box was the one selected, given that the marble is white?

7. In a certain school there are only three sports offered: football, basketball, and soccer.

   - For each of the three sports, the probability is $\frac{1}{10}$ that a student only plays that sport.
   - For any two of the sports, the probability is $\frac{3}{25}$ that a student plays these two sports but not the other.
   - The probability that a student plays all three sports, given that he plays football and basketball is $\frac{1}{3}$.

   What is the probability that a student takes none of the sports, given that he does not take football?

8. There are four sections of Probability, with section I having 40 students, section II having 60 students, section III having 70 students, section IV having 30 students.

   - The probability of getting an A in section I is 0.2.
   - The probability of getting an A in section II is 0.1.
   - The probability of getting an A in section III is 0.3.
   - The probability of getting an A in section IV is 0.05.

   If someone told you that she was in one of the four sections, and that she got an A, what is the probability that

   (a) she was in section I?

   (b) she was in section II?

   (c) she was in section III?

   (d) she was in section IV?

9. A certain math class has some students who took the AP Calculus exam. It is known that

   - 10% of the students did not take the AP exam.
   - 30% scored 4 in the AP exam.
   - the rest of the students scored 5 on the AP exam.
   - 40% of the students who did not take the AP exam failed the class.
   - 10% of the students who scored 4 on the AP exam failed the class.
   - 1% of the students who scored 5 on the AP exam failed the class.

   Given that the student passed the class, what is the probability that the student scored 4 on the AP exam?

10. There are three coins in a box. One is a two headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75% of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two headed coin?