Math 477 Problems for Chapter 2

1. Let E be the set
$$E = \{1, 2, 3, 4, 5, 6, 7, 8\}.$$
Let A, B, C be the subsets
$$A = \{1, 3, 5\}, \qquad B = \{5, 7, 8\}, \qquad C = \{1, 7, 8\}.$$
Compute the following sets
$$A \cap B \cap C, \qquad A^c \cap B \cap C, \qquad A \cap B^c \cap C, \qquad A \cap B \cap C^c, \qquad A^c \cap B^c \cap C^c, \qquad A \cup B \cup C.$$

2. Out of a large group of students, it is found that 22% take both calculus and algebra, and 12% take neither of these two classes. The probability that a student takes calculus exceeds by 0.14 of the probability that a student takes algebra. Determine the probability that a randomly chosen member of this group takes algebra.

3. A survey of 100 students taken over the last year revealed the following:

   - 30 of them smoke,
   - 50 of them drink,
   - 20 of them smoke and drink,
   - 25 go to the gym regularly, and of these, none smokes nor drinks.

   Calculate how many of the 100 students neither smoke, nor drink, nor go to the gym.

4. Out of a class of 100 students,

   - 20 students play football,
   - 10 students play football and soccer,
   - 15 students play football and basketball,
   - 12 students play soccer and basketball,
   - 5 students play football soccer and basketball,
   - 57 students play none of the three sports.

   The number of students who play basketball is 5 less than the number of students who play soccer. How many students play soccer, and how many students play basketball?

5. What is the probability that if you toss a fair coin n times, you would get exactly k tails?

6. There are 10 math majors and 10 CS majors. They have to form teams of two for a competition. If each arrangement is equally likely, what is the probability that no team is mixed, that is, each of the 10 teams of two people consists of either 2 math majors or 2 CS majors.

7. What is the probability that among 30 people, at least two of them have the same birthday? Assume that no one is born on February 29th, and that the remaining 365 days are equally likely.

8. There is a box consisting of 20 red balls, 30 green balls, and 40 yellow balls, and you draw 12 balls without replacing them. What is the probability that you would pick 3 red balls, 4 green balls, and 5 yellow balls?