

## Math 16:642:611 - Topics in Applied Mathematics: Variational Inequalities, Obstacle Problems, and Free Boundary Problems

**Instructor:** Paul Feehan, Department of Mathematics, Rutgers University

**Time:** Tuesdays & Fridays, 10:20am - 12:00pm

**Summary:** The goal of the course is to introduce graduate students to methods for solving variational inequalities, obstacle problems, and free boundary problems and their applications.

Applications will include examples arising in mathematical finance, such as the American-style option pricing problem, and examples drawn from other areas of pure and applied mathematics based on the interests and background of students. Our emphasis will be on the universal applicability of the methods introduced in the course to a wide variety of examples in all areas of pure and applied mathematics.

Topics selected will depend on the interests and backgrounds of the audience, but may include

- ◆ Obstacle problems, variational inequalities, and free boundary problems
- ◆ Application to the American-style option pricing problem
- ◆ Existence, uniqueness, and regularity of solutions to variational inequalities and obstacle problems
- ◆ Optimal regularity of solutions to obstacle problems near the free boundary
- ◆ Stochastic representation of solutions to boundary value and obstacle problems
- ◆ Numerical solution of obstacle problems by finite difference and finite element methods
- ◆ Introduction to viscosity solutions

Students will be polled at the start of the class regarding their interests.

### **Audience:**

- ◆ Doctoral students in pure or applied mathematics as well as doctoral students in operations research and financial engineering or probability and statistics;

- ◆ Master's students in applied mathematics, financial engineering, or mathematical finance with sufficient mathematics background.

The choice of topics and the level of coverage will be carefully adjusted for the audience at the beginning of the semester in order to accommodate different interests (for example, mathematical finance, probability and stochastic analysis, partial differential equations, applied mathematics, or numerical methods) and backgrounds (mathematics, mathematical finance, and other).

### **Prerequisites:**

An undergraduate (or higher-level) course on real analysis covering elementary measure theory (Lebesgue integral), and the concepts of Hilbert spaces and Banach spaces will be useful.

### **Co-requisites:**

A one-semester undergraduate course on partial differential equations (for example, based on the text by Walter Strauss) or a graduate level course on partial differential (for example, based on the text by Lawrence Evans) would be useful, but will not necessarily be assumed.

### **Grading:**

This is an elective course for doctoral students and master's degree students.

I will provide optional homework problems (which may be theoretical or involve use of open source packages for solving partial differential equations and obstacle problems).

### **Textbooks:**

1. P. Feehan, *Lectures on variational inequalities, obstacle, and free boundary problems in mathematical finance*, Rutgers University, Fall 2011.
2. J-F. Rodrigues, *Obstacle problems in mathematical physics*, North-Holland, New York, 1987.

### **Primary references:**

1. A. Bensoussan and J. L. Lions, *Applications of variational inequalities in stochastic control*, North-Holland, New York, 1982.
2. A. Friedman, *Variational principles and free boundary problems*, Dover, New York, 2010.
3. R. Glowinski, J-L. Lions, and R. Trémolières, *Numerical analysis of variational inequalities*, North-Holland, 1981.

4. I. Karatzas and S. E. Shreve, *Methods of mathematical finance*, Springer, New York, 1998.
5. D. Kinderlehrer and G. Stampacchia, *An introduction to variational inequalities and their applications*, Academic, New York, 1980.
6. S. Koike, *A beginner's guide to the theory of viscosity solutions*, MSJ Memoirs, vol. 13, Mathematical Society of Japan, Tokyo, 2004; second edition, 2010, available at [rimath.saitama-u.ac.jp/lab.jp/skoike/book/visbyMizuno.pdf](http://rimath.saitama-u.ac.jp/lab.jp/skoike/book/visbyMizuno.pdf).
7. A. Pascucci, *PDE and martingale methods in option pricing*, Springer, New York, 2011.
8. N. Hilber and O. Reichmann and C. Schwab and C. Winter, *Numerical solution of partial differential equations in financial engineering: finite element and finite difference methods*, Springer, New York, 2013.
9. S. E. Shreve, *Stochastic calculus for finance II: Continuous-time models*, Springer, New York, 2004.
10. J. Steinbach, *A variational inequality approach to free boundary problems with applications in mould filling*, Birkhauser, 2002
11. G. M. Troianiello, *Elliptic differential equations and obstacle problems*, Plenum, New York, 1987.

### **Supplementary references:**

1. M. G. Crandall, *Viscosity solutions: a primer*, in "Viscosity solutions and applications" Lecture Notes in Math., vol. 1660, Springer, 1997, pp. 1-43.
2. M. G. Crandall, H. Ishii, and P-L. Lions, *User's guide to viscosity solutions of second order partial differential equations*, Bull. Amer. Math. Soc. **27** (1992), 1-67.
3. L. C. Evans, *Partial differential equations*, second edition, American Mathematical Society, Providence, RI, 2010.
4. A. Friedman, *Partial differential equations*, Dover, New York, 2008.
5. A. Friedman, *Partial differential equations of parabolic type*, Dover, New York, 2008.
6. D. Gilbarg and N. S. Trudinger, *Elliptic partial differential equations of second order*, Springer, New York, 2001.
7. R. Glowinski, *Lectures on numerical methods for non-linear variational problems*, Springer, 2008.

8. A. Petrosyan, H. Shahgholian, N. Ural'tseva, *Regularity of free boundaries in obstacle type problems*, American Mathematical Society, 2012.
9. Q. Han and F. Lin, *Elliptic partial differential equations*, Courant Lecture Notes, American Mathematical Society, Providence, RI, 2011.
10. N. V. Krylov, *Lectures on elliptic and parabolic equations in Hölder spaces*, American Mathematical Society, Providence, RI, 1996.
11. N. V. Krylov, *Lectures on elliptic and parabolic equations in Sobolev spaces*, American Mathematical Society, Providence, RI, 2008.