Introduction to Complex Analysis of Several Variables

Course Index: Math 523; Spring of 2014

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A function with n complex variables $z \in \mathbf{C}^n$ is said to be holomorphic if it can be locally expanded as power series in z. An even dimensional smooth manifold is called a complex manifold if the transition functions can be chosen as holomorphic functions. Roughly speaking, a Cauchy-Riemann manifold (or simply, a CR manifold) is a manifold that can be realized as the boundary of a certain complex manifold. Several Complex Variables is the subject to study the properties and structures of holomorphic functions, complex manifolds and CR manifolds.

Different from one complex variable, if $n > 1$ one can never find a holomorphic function over the punctured ball that blows up at its center. This is the striking phenomenon that Hartogs discovered about 100 years ago, which opened up the first page of the subject. Then Poincaré, E. Cartan, Oka, etc, further explored this field and laid down its foundation. Nowadays as the subject is intensively interacting with other fields, providing important examples, methods and problems, the basic materials in Several Complex Variables have become mandatory for many investigations in pure mathematics. This class tries to serve such a purpose, by presenting the following fundamental topics from Several Complex Variables.

(a) Holomorphic functions, plurisubharmonic functions, pseudoconvex domains and the Cauchy-Riemann structure on the boundary of complex manifolds

(b) Hörmander's L^2-estimates for the $\bar{\partial}$-equation and the Levi problem

(c) Cauchy-Riemann geometry and subelliptic analysis on CR manifolds

(d) Complex manifolds, holomorphic vector bundles, Kahler Geometry.

The course materials will be largely taken from the following, in particular [5], which I will give the pdf files during the semester:

[1] L. Hormander, *An introduction to complex analysis in several variables*, Third edition, North-Holland, 1990.

[2] James Morrow and K. Kodaira, *Complex Manifolds*, Rinehart and Winston, 1971.

[3] Xiaojun Huang, Lectures on the Local Equivalence Problems for Real Submanifolds in Complex Manifolds, Lecture Notes in Mathematics 1848 (C.I.M.E. Subseries), Springer-Verlag, 2004.

[4] Xiaojun Huang, Subelliptic analysis on Cauchy-Riemann manifolds, Lecture Notes on the national summer graduate school of China, 2007. (to appear)
[5] Xiaojun Huang, Lecture Notes on Several Complex Variables, to appear.
Prerequisites: One complex variable and the basic Hilbert space theory from real analysis