1. Let $M$ be a left $R$-module and let $B = \{ b \in R | bx = 0 \text{ for all } x \in M \}$ be the annihilator of $M$. Show that $B$ is an ideal of $R$. Show that if $C$ is any ideal contained in $B$ then $M$ is a left $R/C$ module under $(a + C)x = ax$.

2. (Schur’s Lemma) Recall that a module $M$ is irreducible (or simple) if the only sub-modules are 0 and $M$.
   a) Show that every irreducible module is cyclic
   b) Show that any nonzero homomorphism from an irreducible module $M_1$ to an irreducible module $M_2$ is an isomorphism. Conclude that if $M$ is irreducible, the endomorphism ring $End_R(M)$ is a division ring.

3. Let $\textbf{Top}$ be the category of all topological spaces, with morphisms the continuous maps. Let $F$ be the forgetful functor to the category of sets. Determine the left adjoint and the right adjoint of $F$.

4. Let $J$ be an index category, and let $C$ be another category. Let $\text{FUNCT}(J, C)$ be the category of functors from $J$ to $C$. Consider the functor $\Delta$ from $C$ to $\text{FUNCT}(J, C)$ which assigns an object $C$ of $C$ to the constant functor in $\text{FUNCT}(J, C)$ with value $C$. Show that the left adjoint of $\Delta$ (if it exists) is the functor assigning to each functor $F$ from $J$ to $C$ the inductive limit of $F$. Similarly, identify the right adjoint of $\Delta$.

5. Hungerford 4.1.7
6. Hungerford 4.2.13
7. Hungerford 4.3.1
8. Hungerford 4.3.3
9. Hungerford 4.3.5