Mathematics 551 **Algebra** **Fall 2006**

Week 2 Generators, orbits and cosets
Sections 1.4 – 1.6

1. Show that the product of all the elements of a finite abelian group is the order two element if a unique such element exists, and is the identity element otherwise. Apply this to the group $(\mathbf{Z}/p\mathbf{Z})^*$ for a prime number p to prove:
   Wilson's Theorem: A natural number n is prime if and only if $(n-1)! \equiv -1 \pmod{n}$.
2. Show that if g is an element of a group and the order of g is n then the order of g^k is n divided by the greatest common divisor of n, k when k is a nonzero integer. Show that the number of generators of the cyclic group which g generates is the number of positive integers m less than n which are relatively prime to n.
3. Show that a finite index subgroup H of a finitely generated group G is finitely generated
4. Hungerford I.3.7
5. Hungerford I.5.19
6. Hungerford I.5.20