1. Show that the product of all the elements of a finite abelian group is the order two element if a unique such element exists, and is the identity element otherwise. Apply this to the group \((\mathbb{Z}/p\mathbb{Z})^*\) for a prime number \(p\) to prove:
Wilson’s Theorem: A natural number \(n\) is prime if and only if \((n - 1)! \equiv -1 \pmod{n}\).

2. Show that if \(g\) is an element of a group and the order of \(g\) is \(n\) then the order of \(g^k\) is \(n\) divided by the greatest common divisor of \(n, k\) when \(k\) is a nonzero integer. Show that the number of generators of the cyclic group which \(g\) generates is the number of positive integers \(m\) less than \(n\) which are relatively prime to \(n\).

3. Show that a finite index subgroup \(H\) of a finitely generated group \(G\) is finitely generated

4. Hungerford I.3.7

5. Hungerford I.5.19

6. Hungerford I.5.20