1. Let $R$ be a commutative ring or a division ring. Suppose that $A, B$ are $n \times n$ matrices with entries from $R$. Show that if $AB = I_n$ then $BA = I_n$. Show that this is not true for a general ring $R$ (using for example Hungerford exercise IV.2.13 or otherwise).

2. Let $I$ be the ideal of $\mathbb{Z}[X]$ generated by 3 and $X$. Show that I is not a direct sum of cyclic $\mathbb{Z}[X]$ modules.

3. Let $S = \mathbb{R}[X]$ and let $M$ be the direct sum of cyclic modules with annihilators $(X - 1)^3, (X^2 + 1)^2, (X - 1)(X^2 + 1)^2, (X + 2)(X^2 + 1)^2$. Find the elementary divisors and invariant factors of $M$.

4. Show that a complex square matrix $A$ is similar to a diagonal matrix if and only if the minimal polynomial has distinct roots.

5. Hungerford 7.2.12
6. Hungerford 7.3.3
7. Hungerford 7.4.7
8. Hungerford 7.4.10
9. Hungerford 7.5.10