1. Let R be a ring. Consider the forgetful functor from the category of (left) R-modules to abelian groups. Determine the right adjoint to this functor (Hint: think about what the morphisms of the abelian group of the R-module R to an abelian group A must be if a right adjoint exists).

2. We say that a category $C$ is additive if
   1. The set of morphisms between two objects is an abelian group, and the maps obtained by composition are abelian group homomorphisms.
   2. There is a zero object 0 such that the morphisms of 0 to itself form the trivial group.
   3. For any pair of objects $X_1, X_2$ there is an object $Y$ and morphisms $p_i : Y \to X_i$ and $j_i : X_i \to Y$ such that $p_i j_i = 1_{X_i}, p_i j_k = 0$ if $i \neq k, j_1 p_1 + j_2 p_2 = 1_Y$.
   Show that the category of modules over a ring is an additive category. Is the category of all commutative rings an additive category?

3. A functor $F$ from an additive category $C$ to an additive category $D$ is said to be additive if for all objects $X, Y$ of $C$ the map that the functor induces from $\text{Mor}(X, Y)$ to $\text{Mor}(FX, FY)$ is a homomorphism of abelian groups. Show that the forgetful functor from modules to abelian groups is additive. Show that if $F$ is additive, then any left or right adjoint of $F$ is also additive.

4. Let $D = \mathbb{Z}[i]$ be the ring of Gaussian integers. Let $K$ be the submodule of $D^3$ generated by $(1, 3, 6), (2 + 3i, -3i, 12 - 18i), (2 - 3i, 6 + 9i, -18i)$. Determine the quotient $D$–module $D^3/K$ as a product of cyclic $D$–modules. Is it a finite set?

5. Hungerford 4.4.11

6. Hungerford 4.5.2

7. Hungerford 4.6.6

8. Hungerford 10.2.3

9. Hungerford 10.2.4