Mathematics 535 Algebraic Geometry Fall 2016

Problem Set 8 (Last revised 11/3/2016)

Let K be an algebraically closed field unless otherwise noted.

1. Let G be an algebraic group.
   a) Show (by using the multiplication in the group) that there is a unique irreducible component $G^0 \subset G$ which contains the identity element of the group.
   b) Show that G^0 is a normal subgroup of G.
   c) Show that for each $g \in G$ the coset $G^0 g$ is an irreducible component of G and that all irreducible components are of this form.
   d) Show that the connected components of G are irreducible, and that G is connected if and only if it is irreducible.

2. Let G be a connected algebraic group acting on a quasiprojective variety X (that is a morphism $G \times X \to X$ is given which is a group action on points). Use Chevalley's theorem to show that each orbit Gx is an irreducible quasiprojective variety.

3. (Harris 6.2) Suppose that the characteristic of K is not 2. Show that a vector $\omega \in \bigwedge^2 V$ is decomposable if and only if $\omega \wedge \omega = 0$ and hence the Grassmannian G(2,V) of 2 planes in V is cut out by quadrics in $\mathbf{P}(\bigwedge^2 V)$.