Problem Set 4 (Last revised 9/29/2016)

1. (Harris 1.27) Show that the images of the maps $\mu, \nu : \mathbf{P}^1 \to \mathbf{P}^2$ given by $\mu[x_0, x_1] = [x_0^3, x_0x_1^2, x_1^3]$ and $\nu[x_0, x_1] = [x_0^3, x_0x_1^2 - x_0^3, x_1^3 - x_0^2x_1]$ are algebraic varieties.

2. (Harris 1.29) Let $\nu_{\alpha,\beta} : \mathbf{P}^1 \to \mathbf{P}^3$ be given by $\nu_{\alpha,\beta}([x_0, x_1]) = [x_0^4 - \beta x_0^3x_1, x_0^3x_1 - \beta x_0^2x_1^2, \alpha x_0^2x_1^2 - x_0x_1^3, \alpha x_0x_1^3 - x_1^4]$. For which α, β is $\nu_{\alpha,\beta}$ a morphism? For these α, β show that the image of this map is a projective variety which is the zero locus of one quadratic and two cubic polynomials.

3. (Harris 2.2) Determine the ring of regular functions on $\mathbf{A}^2 - \{(0, 0)\}$

4. (Harris 2.3) Use 2.2 to show that $\mathbf{A}^n - \{(0, 0, \ldots, 0)\}$ is not isomorphic to an affine variety when $n \geq 2$. What is the situation for $n = 1$?

5. (Harris 2.5) Show that the number of monomials of degree d in $n + 1$ variables is $\binom{n+d}{n}$.