1. (Harris 1.11) For $i = 0, 1, 2$ let $F_i$ be the determinant of the 2 by 2 matrix obtained by removing the $3 - i$ column from

$$\begin{pmatrix} Z_0 & Z_1 & Z_2 \\ Z_1 & Z_2 & Z_3 \end{pmatrix}$$

Show that the intersection of two distinct quadric surfaces $Q_i$ which are the zero set of $F_i$ is the union of a line and the twisted cubic. More generally, for $\lambda = [\lambda_0, \lambda_1, \lambda_2]$ a point in the projective plane let $Q_\lambda$ be the zero set of $\lambda_0 F_0 + \lambda_1 F_1 + \lambda_2 F_2$. Show that for $\mu, \nu$ distinct the intersection of $Q_\mu, Q_\nu$ is the union of the twisted cubic and a line $L_{\mu, \nu}$.

2. (Harris 1.12) Show that any finite set of points on the twisted cubic is in general position.

3. (Harris 1.13) Let $p_i$ be 7 distinct points on a twisted cubic $C$. Show that the zero locus of all quadratic polynomials which vanish at all points $p_i$ is precisely $C$.

4. (Harris 1.15) Let $p_i$ be $kd+1$ distinct points on a rational normal curve $C$ in $\mathbb{P}^d$. Show that any polynomial $F$ of degree $k$ vanishing on the $p_i$ also vanishes on $C$. Show that exercise 1.5 is sharp.

5. (Harris 1.19) Define the cross ratio of 4 ordered points $P_1, P_2, P_3, P_4$ in general position in $\mathbb{P}^1$ to be the image of $P_4$ under the projective transformation sending $P_i$ to $E_i$ for $i = 1, 2, 3$. Show that $P_1, P_2, P_3, P_4$ are projectively equivalent (as an ordered set) to $P_1', P_2', P_3', P_4'$ if and only if the cross ratio of the sets agrees. Generalize this to show that $n+3$ ordered points $P_1, P_2, P_3, \ldots, P_{n+3}$ in general position in $\mathbb{P}^n$ are projectively equivalent to $P_1', P_2', P_3', \ldots, P_{n+3}'$ if and only if the cross ratio of $Q_1, Q_2, Q_3, Q_4$ equals that of $Q_1', Q_2', Q_3', Q_4'$ for $i = 4, \ldots, n+3$ where the points $Q_i$ are the preimage of the points $P_i$ under the map from $\mathbb{P}^1$ given by the unique rational normal curve through the $P_i$ (similarly for $P_i'$).

6. (Harris 1.21) Let $P$ be a point in the projective plane and let $W_P$ be the hyperplane in $\mathbb{P}^5$ defined by the condition that the quadratic polynomial $\lambda_0 Z_0^2 + \lambda_1 Z_1^2 + \lambda_2 Z_2^2 + \lambda_3 Z_0 Z_1 + \lambda_4 Z_0 Z_2 + \lambda_5 Z_1 Z_2$ vanishes at $P$. If $P_i$ are 5 points in $\mathbb{P}^2$ with no 4 collinear show that the hyperplanes $W_{P_i}$ intersect in a single point. Show that this means that there is a unique plane conic curve through the 5 points.