Problem Set 2 (Last revised 9/14/2016)

0. (carryover from problem set 1) Let R be a Boolean ring, that is $r^2 = r$ for all $r \in R$.
   a) Show that R is a commutative ring of characteristic 2 and that all prime ideals are maximal.
   b) Show that spec(R) is a compact Hausdorff space in which every connected component is a point. (a so called Stone space)
   c) Show that if X is compact Hausdorff and totally disconnected then the ring R of continuous functions from X to the field with two elements (considered as a topological space with the discrete topology) is a Boolean algebra R. Show that X is homeomorphic to spec(R).
   d) Show that the constructions above yield an equivalence of categories between Stone spaces with continuous maps, and Boolean algebras with ring homomorphisms. Since a homomorphism $R \to S$ yields by inverse image a map $spec(S) \to spec(R)$ this relation is a duality (introduced by Marshall Stone in 1936).

1. An aerial camera photographs a car traveling along a straight road on flat ground towards a junction. Before the junction there are two warning signs at distances of 4km and 2km from the junction. On the film, the signs are 1cm and 3cm from the junction and the car is 3/7cm from the junction. Use the cross ratio to determine how far the car is from the junction on the ground.

2. (Harris 1.3) Show that any d points in projective space which are not contained in a line are the zero set of a collection of polynomials of degree d-1.

3. (Harris 1.5)Let Γ be a collection of $d \leq kn$ points in general position in projective n space ($k \geq 2$). Show that Γ is the zero set of a collection of homogeneous polynomials of degrees at most k.

4. (Harris 1.6) Let $P_1, \ldots, P_{n+2}$ and $Q_1, \ldots, Q_{n+2}$ be two ordered sets of $n + 2$ points in general position in projective n–space. Show that these varieties are projectively equivalent, and the equivalence can be chosen to preserve the order.