Problem Set 12 (Last revised 12/2/2016)

1. (Harris 13.6) Find the Hilbert function of the Segre variety $\Sigma_{n,m} = \sigma(\mathbf{P}^n \times \mathbf{P}^m) \subset \mathbf{P}^{nm+n+m}$ and verify that the dimension is $n + m$.

2. (Harris 13.8) Determine the arithmetic genus of (i) a pair of skew lines in $\mathbf{P}^3$ (i) a pair of incident lines in either $\mathbf{P}^2$ or $\mathbf{P}^3$ (iii) three concurrent but not coplanar lines in $\mathbf{P}^3$ and (iv) three concurrent coplanar lines in either $\mathbf{P}^2$ or $\mathbf{P}^3$.

3. (Harris 13.9) Consider a plane curve $X \subset \mathbf{P}^2$ of degree d and its image $Y = \nu_2(X) \subset \mathbf{P}^5$ under the quadratic Veronese map. Compare the Hilbert polynomials of the two and observe in particular that the arithmetic genus is the same.

4. (Harris 13.17) Prove a weak form of the Bezout theorem in $\mathbf{P}^2$: if $F, G$ are polynomials of degree $d, e$ on $\mathbf{P}^2$ without common factors such that $F, G$ generate the ideal of their intersection, then the intersection consists of $d \cdot e$ points. Similarly show that if $\Gamma \subset \mathbf{P}^3$ is a complete intersection of surfaces of degrees $d, e, f$ then $\Gamma$ consists of $d \cdot e \cdot f$ points.

5. (Harris 13.18) Find the Hilbert polynomial of a complete intersection in $\mathbf{P}^3$ of surfaces of degrees $d, e$. What is the arithmetic genus of this complete intersection?