1. (Harris 11.11) Find a homogeneous quadratic polynomial $Q(Z_0, \ldots, Z_3)$ and a homogeneous cubic polynomial $P(Z_0, \ldots, Z_3)$ whose common zero locus is a rational normal curve $C \subset \mathbb{P}^3$.

2. (Harris 11.15) Let $X, Y$ be irreducible projective varieties, $f : X \to Z$ and $g : Y \to Z$ surjective maps. Show that the dimension of the fiber product satisfies

$$\dim(X \times_Z Y) \geq \dim(X) + \dim(Y) - \dim(Z).$$

Find an example where strict inequality holds. Show by example that some irreducible components of $X \times_Z Y$ may have strictly smaller dimension.

3. (Harris 11.20) Let $X \subset \mathbb{P}^n$ be an irreducible non-degenerate $k$-dimensional variety. For $l < n - k$ find the dimension of the closure of the locus of $l$-planes meeting $X$ in at least two points. Show by example that no analogous formula exists if we replace two by three, even if we require $l \geq 2$.

4. (Harris 11.21) Let $X \subset \mathbb{P}^n$ be an irreducible non-degenerate $k$-dimensional variety. Show that the plane spanned by $n - k + 1$ general points of $X$ meets $X$ in only finitely many points and use this to compute the dimension of the closure of the locus of $l$-planes spanned by their intersections with $X$ when $l \leq n - k$.

5. (Harris 11.25) Show that if $X \subset \mathbb{P}^n$ is an irreducible curve then the chordal (or secant) variety $S(X)$ is three-dimensional unless $X$ is contained in a plane.

6. (Harris 11.31) Let $X \subset \mathbb{P}^n$ be an irreducible nondegenerate variety of dimension $k$. Use 11.21 to deduce that for any $l \leq n - k$ the general fiber of the secant plane map $s_l : X^{l+1} \to G(l, n)$ is finite.

7. (Harris 11.39) Show that if $m > n$ there does not exist a nonconstant regular map $\phi : \mathbb{P}^m \to \mathbb{P}^n$ by using 11.38. Hint: First try the case $n=1$ and look at dimensions of inverse images of a hyperplane and a point not on it.

8. (Harris 11.41) Find the dimension of the flag variety $F(a_1, \ldots, a_n)$ in general.