Problem Set 6A (Last revised 11/21/2008)

7.28 Generalize 7.11 by showing that $\mathbf{P}^m \times \mathbf{P}^n$ is birational with $\mathbf{P}^{m+n}$ via the map

$$\phi([Z_0, \ldots, Z_n], [W_0, \ldots, W_n]) = [Z_0W_0, Z_1W_0, \ldots, Z_mW_0, Z_0W_1, \ldots, Z_0W_n].$$

What is the graph of this map? Describe the map in terms of blowing up and blowing down.

11.11 Find a homogeneous quadratic polynomial $Q(Z_0, \ldots, Z_3)$ and a homogeneous cubic polynomial $P(Z_0, \ldots, Z_3)$ whose common zero locus is a rational normal curve $C \subset \mathbf{P}^3$.

11.39 Show that if $m > n$ there does not exist a nonconstant regular map $\phi : \mathbf{P}^m \to \mathbf{P}^n$ by using 11.38.

11.44 Let $X \subset \mathbf{P}^n$ be an irreducible k-dimensional variety. Find the dimension of the universal $(n-l)$-plane section $\Omega^{(l)}(X)$ and show that it is irreducible.

12.17 Observe that a curve of type $(1, d-1)$ on a smooth quadric $Q \subset \mathbf{P}^3$ is a rational curve of degree d, that is the image of $\mathbf{P}^1$ under a map to $\mathbf{P}^3$ given by a quadruple of polynomials of degree d on $\mathbf{P}^1$, or equaivalently a projection of the rational normal curve in $\mathbf{P}^d$. Use a count of parameters to show that for $d \geq 5$ the general rational curve of degree d is not a curve of type $(1, d-1)$ on a quadric.